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SIMPLIFIED AND DIRECT DESIGN METHODS FOR VIBRATION ISOLATION STRUCTURES INVOLVING TWO MASSES AND TWO DEGREES OF FREEDOM.

SIMPLIFIED AND DIRECT DESIGN METHODS FOR VIBRATION ISOLATION STRUCTURES INVOLVING TWO MASSES AND TWO DEGREES OF FREEDOM

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INTRODUCTION

The analysis and design of structures with more than one degree of freedom and subjected to dynamic loads, involves the simultaneous solution of second order differential equations which can be tedious. Adding to the complexity of the problem is the fact that the frequency composition of the forcing function must be considered with respect to each natural mode of vibration since there are as many natural modes of vibration as there are degrees of freedom. Often, especially in the case of the design of a structure to resist dynamic loads, the problem is reduced to a long and tedious trial and error process at best.

A design problem involving dynamic loads which is frequently encountered is that of vibration isolation structures which may be represented by two masses and two degrees of freedom. Some typical examples will be given later. Using a two story shear building as the basic model for such structures and with the aid of a high speed electronic digital computer, the author has obtained simultaneous solutions of the governing differential equations of motion for each mass for various stiffness, mass, and frequency ratios. The excitation

of the system was applied in two ways. First the model was subjected to periodic base motion to simulate ground motion caused by earthquakes. Secondly, the model was subjected to a periodic forcing function applied at the second story of the model to simulate vibrations caused by rotating or reciprocating machinery. The response of the model in each of the above cases is presented in the form of curves and nomographs (charts). The theory and the methods used in obtaining these charts and their application to the design of vibration isolation structures, involving two masses and two degrees of freedom, follows. The author wishes to emphasize the fact that the intent and purpose of this thesis is to provide simplified design procedures for vibration isolation structures which can be reasonably represented by a two degree of freedom system. Information relative to previous work in this area, specifically with regard to design of structures to resist earthquake forces may be found in references 5 and 6. The design method presented hereinafter is an extension to the work mentioned above.

NOMENCLATURE

<u>General</u>

A	Area; Constant
B,C	Constants
E	Young's Modulus
F	Force
I	Moment of Inertia
K	Linear Spring Stiffness
L	Length
M	Equivalent Mass
N	Mass of a Rotating Body
F(t)	Periodic Force
Q	Bending Moment
X(t)	Periodic Displacement
Xm	Maximum Absolute Displacement of X(t)
X	Displacement
X''	Acceleration
X'	Velocity
Y,Z	Maximum Absolute Displacement
W	Concentrated Load
a	Amplitude Ratio
cu C_u	Coefficient of Elastic Uniform Compression
cp C_p	Coefficient of Uniform Compression
g	Acceleration due to Gravity
p	Natural Circular Frequency
r	Distance from axis or rotation to Mass center of m

NOMENCLATURE (Con't)

t Time

w Frequency of Forcing Function

$\overline{w}$ Distributed Load

y Static Deflection due to Distributed Load

Δ Static Deflection due to Concentrated Load

Concrete

b Width of section

d Distance from extreme fiber to steel reinforcement

f'_c Maximum compressive stress allowed in concrete

jd Distance between resultants of the tensile and compressive forces on the section (expressed as a fraction of d)

kd Distance from extreme compressive fiber to neutral axis (expressed as a fraction of d)

The subscript 1 or 2 or the numeral 1 or 2 following any of the above symbols refer to mass 1 or 2 respectively. Mass 1 or mass 2 refers to the equivalent mass of the 1st or 2nd story respectively of the shear building model.

PART I. SOLUTION OF GOVERNING DIFFERENTIAL EQUATIONS OF MOTION OF A TWO DEGREE OF FREEDOM SYSTEM

A. GENERAL

The multistory building is perhaps the most illustrative example of a structure which moves with many degrees of freedom. Remembering that a rigid body has six degrees of freedom, the motion of each floor could be described by 6 differential equations and therefore the motion of a two story building would require 12 differential equations of motion. However, the vertical stiffness of a building is much greater than the horizontal stiffness so vertical translational motion and rotational motion about horizontal axes can be disregarded. A further simplification can be made if the structure is symmetrical because the number of equations required reduces to three sets of two equations each with each set being independent of the others. They are a set in X_1 and X_2, a set perpendicular to X_1 and X_2 in the horizontal plane, and a set about the vertical axes of each floor.

The model that is to be used is called a shear building, see figure 1. By neglecting, the rotation of a horizontal section at the level of the floors, the building in horizontal deflection,

X_1 and X_2 directions, will act as a cantilever beam subjected to shear forces only. For this reason this particular model is often called a shear building. Three basic assumptions must be made to insure that only horizontal deflections will occur when shear type forces are applied to the structure. First, the distributed mass of the girders and columns is concentrated at the floors. A second assumption fixes the joints between the columns and the girders against rotation by making the girders infinitely rigid as compared to the columns. The third assumption is that the deformation of the structures is independent of axial forces in the columns so that the girders remain horizontal when displaced horizontally.

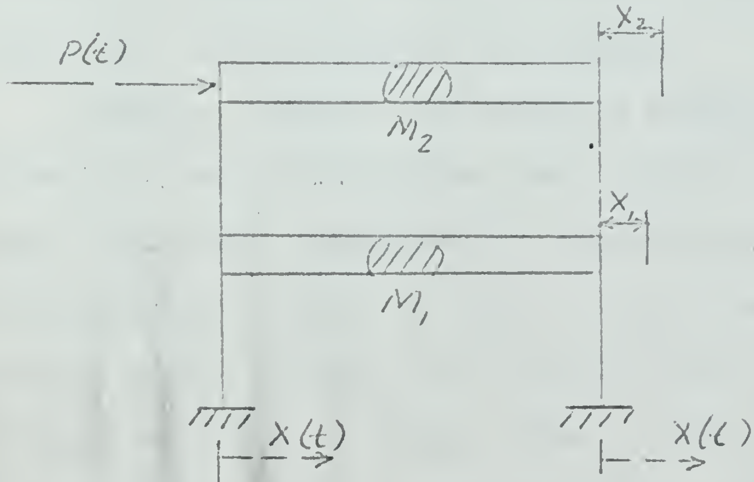

Figure 1

In the design of vibration isolation structures the designer is generally confronted with two types of problems. One situation may require the isolation of piece of equipment from external vibrations such as periodic foundation movements or earthquakes. The other situation may be the isolation of unbalanced rotating machinery, such as a rock crusher from its foundation. Accordingly, in the solution of the governing differential equation of motion of the two story shear building model, two conditions of excitation will be used; periodic base motion and a periodic forcing function applied horizontally to the second floor.

The governing differential equation of motion is derived from Newton's Second Law of Motion. In as much as it can be found in any basic textbook on dynamics, such as Jacobsen and Ayre [1], Rogers [2], its derivation is omitted.

Finally, because the absolute maximum displacements and accelerations most probably will occur during the transient era except when a resonant condition exists, as will be shown later and because design criteria are normally based on the absolute maximums, damping, which usually exists, will be omitted in the solution of the equations.

Before proceeding with his approach, the author wishes to point out that several methods are available for the investigation of multiple degree of freedom structures. An accurate and often used approach for structures whose free vibration can be resolved into normal modes of vibration is the modal method, see Harris and Crede [6]. As an example, equations for the free vibration response of such structures can be written directly and in general terms as

$$y = \sum C_i \, \phi_i \quad (x) \sin p_i \, t$$

where

y = relative displacement at point x

$\phi_i(x)$ = shape of the i th mode of vibration

p_i = natural frequency of the i th mode

c_i = participation factor of the i th mode

However, to use this method the mode shapes and their respective frequencies must be obtained first.

In as much as the author is dealing only with a two degree of freedom system in a design situation he feels that the use of the modal method would not further simplify the problem. Having obtained the equations of motion by the application of D'Alembert's Principle to free bodies, the solution of these equations as obtained by the author's approach is essentially the same as that of the modal method.

B. EXCITATION BY PERIODIC BASE MOTION

Assuming that the base of the model in figure 1 is subjected to a periodic displacement, $X(t)$, the governing differential equations of motion for each mass can be written with the aid of the free bodies shown in figure 2.

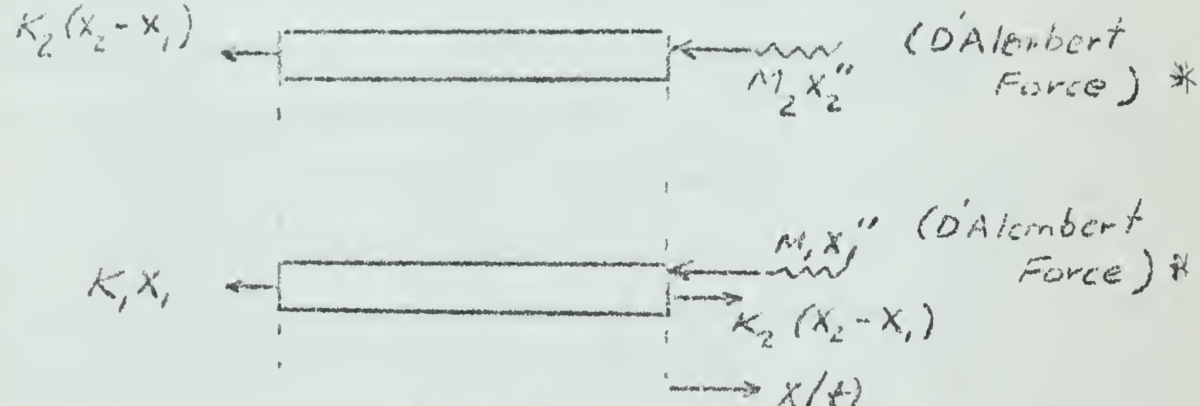

figure 2

Summing forces in the horizontal direction gives,

$$M_1X_1'' + K_1X_1 - K_2(X_2 - X_1) = K_1X(t) \qquad \text{(a)}$$

$$M_2X_2'' + K_2(X_2 - X_1) = 0 \qquad \text{(b)}$$

Letting $X(t) = X_m \sin wt$, where X_m is the maximum displacement, and rearranging, equations (a) and (b) become

$$M_1X_1'' + (K_1 + K_2)X_1 - K_2X_2 = K_1X_m \sin wt \qquad \text{(c)}$$

$$M_2X_2'' - K_2X_1 + K_2X_2 = 0 \qquad \text{(d)}$$

*D'Alembert's Principle: The resultant of the effective forces for all particles of a body, if reversed and assumed to act on the body with the external forces, will hold the body in equilibrium.

A particular solution, namely a stationary harmonic solution of the single frequency w will be obtained first. If $X_1 = Z_1 \sin wt$ then upon differentiating with respect to t $X_1'' = -w^2 Z_1 \sin wt$ and in a like manner if $X_2 = Z_2 \sin wt$ then $X_2'' = -w^2 Z_2 \sin wt$ where Z_1 and Z_2 are transient maximum displacements. Substituting into equations (c) and (d) and collecting terms

$$(K_1 + K_2 - M_1 w^2)\, Z_1 - K_2 Z_2 = XmK_1 \qquad \text{(e)}$$

$$-K_2 Z_1 + (K_2 - M_2 w^2)\, Z_2 = 0 \qquad \text{(f)}$$

From the simultaneous solution of equations (e) and (f)

$$Z_1 = \frac{XmK_1\left[\dfrac{K_2}{M_1 M_2} - \dfrac{w^2}{M_1}\right]}{w^4 - w^2\left[\dfrac{K_1 + K_2}{M_1} + \dfrac{K_2}{M_2}\right] + \dfrac{K_1 K_2}{M_1 M_2}} \qquad \text{(g)}$$

$$Z_2 = \frac{\dfrac{K_1 XmK_2}{M_1 M_2}}{w^4 - w^2\left[\dfrac{K_1 + K_2}{M_1} + \dfrac{K_2}{M_2}\right] + \dfrac{K_1 K_2}{M_1 M_2}} \qquad \text{(h)}$$

A homogeneous solution is obtained by considering the motion of the model during free vibration. Equations (a) and (b) are applicable if the

right hand side is set equal to zero. If $X_1=Y_1$ cos pt, $X_2=Y_2$ cos pt, $X_1'' = -p^2\ Y_1$ cos pt, and $X_2''= -p^2Y_2$ cos pt where Y_1 and Y_2 are maximum displacements and p is the natural circular frequency of a particular mode, equations (i) and (j) result after appropriate substitutions are made and terms are collected.

$$-M_1Y_1p^2 + Y_1(K_1+K_2) - K_2Y_2 = 0 \qquad (i)$$

$$-M_2Y_2p^2 - Y_1K_2 + K_2Y_2 = 0 \qquad (j)$$

In order to obtain an equation for the natural circular frequency in terms of known quantities let the amplitude ratio $a=\frac{Y_2}{Y_1}$ then equations (i) and (j) become

$$a=\frac{K_1 + K_2 - M_1\ p^2}{K_2} \qquad (k)$$

$$a=\frac{K_2}{K_2-M_2p^2} \qquad (l)$$

The frequency equation is now obtained by eliminating a from (k) and (l)

$$p^4 - p^2\left[\frac{K_1\ +K_2}{M_1} + \frac{K_2}{M_2}\right] + \frac{K_1K_2}{M_1M_2} = 0$$

The two positive roots obtained from this equation are the natural circular frequencies of the first (lower) and second modes namely,

$$p_1^2 = \left[\frac{K_1 + K_2}{2M_1} + \frac{K_2}{2M_2}\right] - \sqrt{\left[\frac{K_1+K_2}{2M_1} + \frac{K_2}{2M_2}\right]^2 - \frac{K_1K_2}{M_1M_2}} \quad (m)$$

$$p_2^2 = \left[\frac{K_1+K_2}{2M_1} + \frac{K_2}{2M_2}\right] + \sqrt{\left[\frac{K_1+K_2}{2M_1} + \frac{K_2}{2M_2}\right]^2 - \frac{K_1K_2}{M_1M_2}} \quad (n)$$

By substituting p_1 or p_2 into either equation (k) or (l) the amplitude ratios a_1 or a_2 respectively may be obtained.

Thus

$$a_1 = \frac{K_1+K_2-M_1p_1^2}{K_2} \quad (o)$$

$$a_2 = \frac{K_1+K_2-M_1p_2^2}{K_2} \quad (p)$$

A complete solution of the equations of motion is obtained by adding to the homogeneous solution the particular solution. The forms of the resulting equations are

$$X_1 = A_1 \cos p_1t + B_1 \sin p_1t + A_2 \cos p_2t + B_2 \sin p_2t + Z_1 \sin wt \quad (q)$$

$$X_2 = a_1A_1 \cos p_1t + a_1B_1 \sin p_1t + a_2A_2 \cos p_2t + a_2B_2 \sin p_2t + Z_2 \sin wt \quad (r)$$

where A and B are arbitrary constants that make the equations satisfy the following initial conditions. Assuming that the system is initially at rest when $t=0$, then $X_1=X_2=X_1'=X_2'=0$. Therefore at $t=0$

$$X_1 = A_1 + A_2 = 0$$

$$X_2 = a_1 A_1 + a_2 A_2 = 0$$

Since $a_1 \neq a_2 \neq 0$, $A_1 = A_2 = 0$. Differentiating equations (q) and (r) once with respect to t the following equations result when $t=0$:

$$X_1' = p_1 B_1 + p_2 B_2 + wZ_1 = 0$$

$$X_2' = a_1 p_1 B_1 + a_2 p_2 B_2 + wZ_2 = 0$$

A simultaneous solution gives

$$B_1 = -\frac{w}{p_1}\left[\frac{Z_2 - a_2 Z_1}{a_1 - a_2}\right]$$

$$B_2 = \frac{w}{p_2}\left[\frac{Z_2 - a_1 Z_1}{a_1 - a_2}\right]$$

Substituting into equations (q) and (r)

$$X_1 = -\frac{w}{p_1}\left[\frac{Z_2 - a_2 Z_1}{a_1 - a_2}\right] \sin p_1 t + \frac{w}{p_2}\left[\frac{Z_2 - a_1 Z_1}{a_1 - a_2}\right] \sin p_2 t + Z_1 \sin wt \qquad \text{(s)}$$

$$X_2 = -a_1 \frac{w}{p_1}\left[\frac{Z_2 - a_2 Z_1}{a_1 - a_2}\right] \sin p_1 t + a_2 \frac{w}{p_2}\left[\frac{Z_2 - a_1 Z_1}{a_1 - a_2}\right] \sin p_2 t + Z_2 \sin wt \qquad \text{(t)}$$

where Z and a are given by equations (g), (h) and (o), (p) respectively.

C. EXCITATION BY PERIODIC FORCE APPLIED TO SUPERSTRUCTURE

By applying a forcing function P(t) to the second floor of the model the resulting differential

equations of motion are

$$M_1X_1'' + K_1X_1 - K_2(X_2 - X_1) = 0 \qquad \text{(aa)}$$

$$M_2X_2'' + K_2(X_2 - X_1) = P(t) \qquad \text{(bb)}$$

Applying the same procedure used in subpart B the particular solution of equations (aa) and (bb) result in

$$Z_1 = \frac{F\dfrac{K_2}{M_1M_2}}{w^4 - w^2\left[\dfrac{K_1+K_2}{M_1} + \dfrac{K_2}{M_2}\right] + \dfrac{K_1K_2}{M_1M_2}} \qquad \text{(gg)}$$

$$Z_2 = \frac{F\left[\dfrac{K_1+K_2}{M_1M_2} - \dfrac{w^2}{M_2}\right]}{w^4 - w^2\left[\dfrac{K_1+K_2}{M_1} + \dfrac{K_2}{M_2}\right] + \dfrac{K_1K_2}{M_1M_2}} \qquad \text{(hh)}$$

where F is a force due to the reaction of a mass $\bar{M}$ rotating about and axis at a distance $\bar{r}$ from its mass center: $F = \bar{M}\bar{r}\, w^2$

The complete solution is obtained by merely substituting into equations (s) and (t) the values of Z_1 and Z_2 as given by equations (gg) and (hh). The expressions for the amplitude ratios and the natural circular frequencies are the same as those derived previously. The constants A and B are also unchanged. For more detailed coverage of the above material the reader is invited to see Jacobsen and Ayre [1] (Chapter 7).

PART II. DEVELOPMENT OF CHARTS

A. GENERAL

Having obtained a complete and general solution to the governing differential equations of motion, the next step involved the use of a high speed electronic digital computer to obtain many specific solutions to the equations. The data generated by the computer was made a function of as few parameters as possible in order to reduce the amount of computer effort required and ultimately, to keep the desired charts simple and the number to a minimum. Each floor of the model has associated with it a mass and a stiffness. The system as a whole has two mode frequencies and a frequency of the forcing function. Therefore a total of 7 different parameters would have been required. If each parameter were given 10 values each, then the number of computations required to obtain an answer for every combination would have been 10^7 which amounts to approximately 150,000 printed pages (11 inches by 13½ inches) of computer output.

To reduce the output and to obtain data that was concise and yet useful, the input to the computer

was reduced to 4 parameters, namely, K_2/K_1, M_2/M_1, w/p, and K_1/M_1. Note that all of the above ratios except the last one are dimensionless. Initially, the author attempted to rearrange the equations of motion so that all terms would be in dimensionless form. However, because of the complex nature of the equations a completely dimensionless set could not be readily obtained. It was therefore decided arbitrarily to let M_1 equal unity and then relate the stiffnesses and M_2 to M_1.

The range of reasonable values for the ratio K_1/M_1 was determined by using Raleigh's well known Static Deflection Method in which the natural circular frequency of a system is given in Jacobsen and Ayre 1 (2-5) as

$$p^2 = g \frac{\int \bar{w} y\,dx + \sum W_i \Delta_i}{\int \bar{w} y^2 dx + \sum W_i \Delta_i^2} \qquad \text{(u)}$$

where g = acceleration due to gravity

Δ = static deflection due to concentrated load

W = concentrated load

y = static deflection due to distributed load

$\bar{w}$ = distributed load

Figure 3(a) shows the deflected shape of the first story of the shear building model which has been rotated 90 degrees.

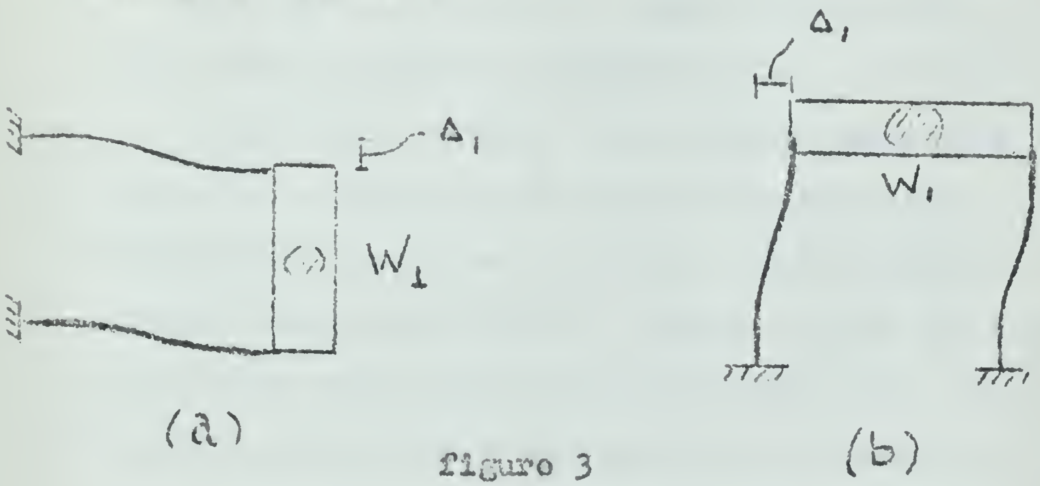

figure 3

The static deflection Δ_1 is caused by gravity acting on the equivalent mass, M_1, and where $W_1=M_1g$. If the deflected shape in figure 3(a) is maintained until the model is returned to the upright position as shown in figure 3(b) and then released, the frequency of vibration will be given by equation (u). Because only one concentrated load is involved, equation (u) reduces to

$$p^2 = g \frac{W_1 \Delta_1}{W_1 \Delta_1^2} = \frac{g}{\Delta_1}$$

A reasonable limiting value of Δ_1 is 1 inch and $p^2 = 386/1 = 386$ (say 400), remembering also that p^2 equals K/M. Therefore a range of ratios of K_1/M_1 would be 400 to say 40000 ($\Delta_1 = .01$ inch).

Appendix A contains the programs that were used in the repetive solution of the governing differential equations of motion. The compiler language used was FORTRAN. Program A-1 is for the base motion case and A-2 for the other case. Both programs are identical except for the subroutine which generates the maximum transient desplacements, Z.

The required input is as follows:

N1 through N4 - the number of values for each parameter

RATIO, STIF, SLUG, WBYD - the four parameters that are respectively K_1/M_1, K_2/K_1, M_2/M_1, and w/p_1

The programs generated the following data for all combinations of input:

p_1= frequency of fundamental natural mode

p_2= frequency of seconinatural mode

X_1= displacement of mass 1

X_2= displacement of mass 2

As mentioned earlier, design criteria and procedures require that the design of a structure be based on maximums such as the maximum deflection or acceleration anticipated for a given set of conditions. In actuality the desired response charts are maximax spectra curves for the various values of the four parameters. There are several ways in which the maximax values of displacement may be obtained in solving equations (s) and (t). The mathematical approach would be to differentiate equations (s) and (t) once with respect to t setting these derivatives equal to zero and then solving for t's which when substituted into the original equations would give the maximum values of displacement. A numerical approach would be to substitute small values of t successively in equations (s) and (t) and by plotting displacement vs. time obtain the maximum displacement. It should be noted that both methods are laborious even when using a computer because the procedures described must be repeated for each combination of stiffness, mass and frequency ratios used.

Again, in the interest of saving computer time and storage space the author chose a practical approach and let sin pt = sin wt = 1 and added the

absolute values of the three groups of quantities in their respective equations to obtain the maximum displacements.

$$X_1 = \frac{w}{p_1}\,\frac{Z_2 - a_2 Z_1}{a_2 - a_1} + \frac{w}{p_2}\,\frac{Z_2 - a_1 Z_1}{a_1 - a_2} + Z_1 \quad \text{(ss)}$$

$$X_2 = a_1 \frac{w}{p_1}\,\frac{Z_2 - a_2 Z_1}{a_1 - a_2} + a_2 \frac{w}{p_2}\,\frac{Z_2 - a_1 Z_1}{a_1 - a_2} + Z_2 \quad \text{(tt)}$$

A comparison of values of X_1 and X_2 shows that the author's approach is valid.

CONDITIONS		NUMERICAL	PRACTICAL
$K_2/K_1 = 1$	X_1	7.46	7.60
$M_2/M_1 = 1$			
$K_1/M_1 = 10000$	X_2	11.51	11.58
$w/p_1 = 0.9$			
$K_2/K_1 = 1$	X_1	8.58	8.77
$M_2/M_1 = 0.5$			
$K_1/M_1 = 4000$	X_2	11.67	11.92
$w/p_1 = 0.9$			

As noted in Harris and Crede [6] (vol 3) for two degree of freedom structures, the author's approach overestimates the true maximum by approximately 5 or 10 percent if the stiffness ratio is very large or very small. If the stiffness ratio is near unity, the method presented hereinafter may be very inaccurate because damping has been neglected. However, the inaccuracy is on the con-

servative side and improvements to the original design, obtained by the procedures set forth hereinafter, can be effected by a more accurate analysis.

B. FREQUENCY CHARTS

It can be seen from equations (m) and (n) that the frequency of a two mass system is a function of four independent variables, namely each mass and

its respective stiffness. Setting M_1 equal to unity the number of variables reduces to three by using the ratios, K_1/M_1, K_2/K_1, and M_2/M_1. In the computer program $K_1 = K_1/M_1$, $K_2=K_2/K_1$ after K_1 is premultiplied by K_1/M_1 and $M_2=M_2/M_1$. As will be seen later the frequency of the fundamental natural mode is an entering parameter of the displacement curves and a rapid determination of this frequency is required. Therefore, and because of the number of variables involved it was decided that a nomograph would be the best method for displaying equation (m) graphically. It was further decided that the frequency scale should be a linear scale to facilitate reading.

Having computed the frequency for each combination of K_1/M_1, stiffness ratio and mass ratio shown on Chart 1, Appendix B, the nomograph in chart 1 was obtained by trial and error. First the pivot line was drawn vertically at the center of the page. Next a vertical frequency line was drawn an arbitrary distance to the right of the pivot line. For the first try the frequency line was drawn at the right edge of the page. A suitable linear scale for the frequency line was then selected. The selection of vertical lines for the stiffness and ratio and mass ratio was arbitrary also. The line

representing K_2/K_1 was drawn at the left edge of the page and the line representing M_2/M_1 at the left quarter point. The line representing K_1/M_1 was determined by construction because the slope of this line could not be arbitrarily set at the vertical. The midpoint of the pivot line was assumed to correspond to the pivot point for $K_2/K_1=C_1$, $M_2/M_1=C_2$. Construction lines were drawn from this point to the points on the frequency scale corresponding to C_1 and C_2 and the different values of K_1/M_1. A second pivot point an arbitrary distance below the first was assumed to correspond to $K_2/K_1=C_1$, $M_2/M_1=C_3$. Construction lines were drawn between this point and points on the frequency scale corresponding to C_1 and C_3 and the different values of K_1/M_1. A line connecting the intersection of the respective construction lines is the line that represents K_1/M_1. Thus by a "bootstrapping" process the values of the stiffness and mass ratios were plotted on their respective lines. Although the process appears to be primitive and unsophisticated, adjustments were made after each step so that the final position of the lines of the nomograph was apparent at the completion of the first trial. Only two subsequent tries were

required to refine Chart 1. Charts 2 and 3, Appendix B are merely an extension of Chart 1.

Charts 4 and 5 of Appendix B gives the frequency of the second natural mode. A similar trial and error process was used in deriving the nomographs with one simplification. It was found in the computations that the frequency of the second natural mode varied as the square root of the ratio K_1/M_1. The multiplier line actually represents the frequency of the second natural mode at various stiffness and mass ratios at $K_1/M_1=1000$.

C. DISPLACEMENT CHARTS

Appendix C contains the displacement charts for a two mass system. The curves on each chart were obtained by plotting the displacement as a function of the ratio of the forcing function frequency to the frequency of the fundamental natural mode for specific values of stiffness and mass ratios. Each chart is for a specific or inclusive value of stiffness ratio with a family of curves for the range of mass ratios (1/50 to 50/1).

Charts 1 through 12, Appendix C are for the case of periodic base motion. The units of dis-

placement are consistent with the units of the periodic displacement. It is interesting to note that the displacement of both masses in this case does not vary if K_1/M_1 is varied. Looking at equations (g) and (h) note that the maximum force applied to the system by the periodic displacement of the base is XmK_1. Since M_1 is unity the force applied to the system is directly proportional to K_1 and if K_1 is small the corresponding value of the forcing function is small and if K_1 is large the corresponding value of the forcing function is large. For this reason the displacement does not vary with the K_1/M_1 ratio.

Chart 13 through 24, Appendix C are for the case of the forcing function applied to the 2nd floor. In this case the displacement is directly proportional to F which is independent of K_1/M_1. Accordingly the K_1/M_1 ratio must be taken into consideration in determining the displacements of the system.

The multiplier values in Charts 13 through 24, Appendix C are displacements for $K_1/M_1=400$. The units are consistent with the units used for the stiffnesses and masses of the system.

PART III. DESIGN APPLICATIONS

A. GENERAL

To illustrate the use and application of the charts in Appendix B and Appendix C, solutions to two design problems are presented.

It is assumed that the structure and equipment can be represented by a two mass system such as the model described in Part I in which M_1 equals an equivalent mass of the foundation, K_1 is a function of the elastic properties of the soil, M_2 equals an equivalent mass of the equipment and supporting framework, and K_2 equals the lateral stiffness of the supporting framework. In the first example equipment on a suitable support on an isolated foundation is subjected to periodic base motion and the problem is to design the structure so that the acceleration of the equipment is limited to a specific value.

In the second example it is assumed that vibration is induced in a piece of equipment by an unbalanced ~~of~~ rotating weight and the problem is to design the equipment so that the deflection of a component is limited to a specific value.

Before proceeding with the examples it is necessary to discuss a method of obtaining an equivalent stiffness of the soil mass supporting the foundation. As noted earlier this equivalent stiffness is represented by K_1 in the shear building model. In the discussion to follow it is assumed that the reader has knowledge of the basic terms and fundamentals of soil mechanics.

The static bearing capacity of a particular soil mass is determined by a series of standard load tests in which a number of concentrated loads are transferred to the soil by a rigid plate, normally less than 30 inches in diameter.

Data observed during these tests are plotted giving curves whose ordinates are settlement in inches and whose abcissas are pressure intensities in tons per square foot. These curves are called loading test curves and for many soils a typical curve initially would be a straight line followed by a continuously steepening curve. In the straight line portion of the curve the ratio between the pressure intensity and the settlement is constant and is often called the static coefficient of settlement which is denoted by c_p: (See Taylor [3] (Chapter 19)).

$$c_p = \frac{\text{pressure intensity at proportional limit*}}{\text{total settlement at proportional limit}}$$

A second coefficient which will be used later to obtain an equivalent stiffness of a given soil mass is called c_u, the static coefficient of elastic uniform compression. It is always larger than c_p since the elastic settlement of a foundation is always smaller than its total settlement: (See Barkan [4]).

$$c_u = \frac{\text{pressure intensity at proportional limit*}}{\text{elastic portion of total settlement at proportional limit}}$$

The units of c_p and c_u are usually given in tons per square foot per inch of settlement or in pounds per cubic inch. The determination of suitable values of the coefficient of elastic uniform compression for any given soil is difficult in as much as c_u depends on a large number of variables whose effects are not easily evaluated. The theory and derivation of equations for c_u are covered in detail in Barkan [4]. For design purposes, Barkan [4] has compiled, from static and dynamic test data, a table of representative values of c_u for different soil groups.

* point at which curve deviates from a straight line

TABLE I

SOIL GROUP CATEGORY	SOIL GROUP	PERMISSABLE STATIC LOAD tons/ft^2	c_u tons/ft^2/in
I	Weak soils(clays and silty clays with sand, in a plastic state; clayey and silty sands; also soils in category II and III with laminae of organic silt and of peat)	up to 1.5	up to 7.6
II	Soils of medium strength (clays and silty clays with sand, close to the plastic limit; sand)	1.5 to 3.5	7.6 to 12.7
III	Strong soils(clays and silty clays with sand, of hard consistency; gravels and gravelly sands; loess and loessial soils)	3.5 to 5	12.7 to 25.4
IV	Rocks	greater than 5	greater than 25.4

The values given in Table 1 are by no means exact but are sufficiently representative of actual values to give reliable design results. Using Table I, values of dynamic vertical and lateral stiffness of a given soil mass can be obtained quickly. Barkan [4] found that the vertical dynamic stiffness is equal to the static c_u multiplied by the area of the base of the foundation and further that the lateral dynamic stiffness is equal to approximately one-half the vertical stiffness.

B. DESIGN EXAMPLE ONE: A BASE MOTION PROBLEM

A A satellite tracking camera and its associated electronic equipment is to be installed in a southern California desert (earthquake zone). The combined weight of the camera and electronic components is 100 tons all of which must be mounted on a girder platform 3 feet above ground surface. The camera is sensitive to vertical and horizontal accelerations and it is desired to design the supporting structure so that the maximum acceleration of the equipment during an earthquake is .26g. The size and configuration of the supporting girders specified by the manufacturer are as shown in figure 4.

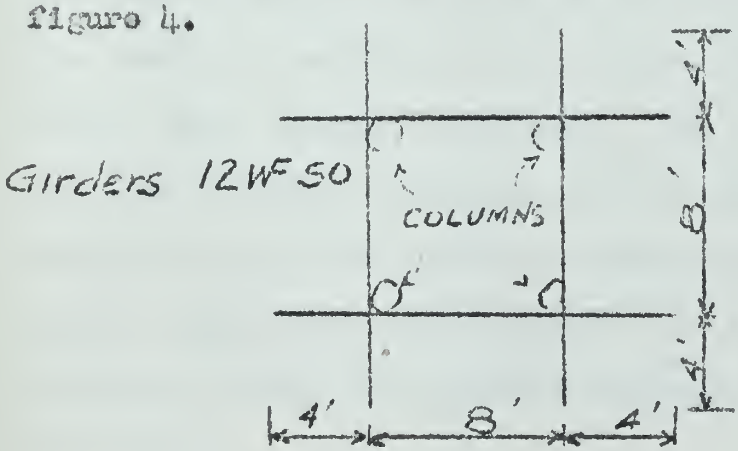

figure 4

All equipment will be fastened to the girders and the loading is such that all columns will carry equal loads with little or no bending.

Wind loads will be neglected as a matter of

simplification. Assume that the strongest recorded earthquake in the vicinity was the El Centro earthquake of May 18, 1940, with a maximum ground acceleration of .33g, a maximum ground velocity of 13.7 inches per second, and a maximum ground displacement of 8.33 inches. The frequency of the displacement was about one-half cycle second, see Blume, Newmark, and Corning [5]. During an earthquake the ground moves at random in all directions and the quantities recorded above are assumed to be representative of horizontal as well as vertical ground motion.

A good and sometimes the only approach to a structural dynamics problem is to first design the structure so that it will at least be strong enough under static conditions. Then, having obtained a static solution, analyze this first design to see if it meets the dynamics criteria. Making appropriate adjustments so that the structure satisfies static and dynamics criteria completes the design.

The static solution requires the design of the columns and the foundation.

Column Design

Load on each column:	25 tons
Estimated allowable compressive steel stress:	18 kips/in^2
Area of column required:	2.78 in^2

Use = 4.0 inch steel pipe

Area = 3.174 in^2

Moment of Inertia $\doteq$ 7.233 in^4

radius of gyration $\doteq$ 1.51 in

weight $\doteq$ 10 lbs/ft

Foundation Design

Load on foundation:

Live load	200 kips
Foundation (10% of LL)	20 kips
Columns and Girders	4 kips
Total	224 kips

From Table 1 the allowable bearing capacity of sand: - use	1.0 tons/ft^2
Area of foundation required:	75 ft^2
Use 10 ft square foundation since columns are spaced 8 ft apart: - Area	100 ft^2
Net soil pressure (weight of foundation not used): -	$\frac{204 \text{ kips}}{100 \text{ ft}^2}$ = 2.04 kips/ft^2

Maximum bending moment
in foundation(at center):

$$\frac{12}{}\ (102x4) - (2.04x10x5x2.5) = 1835 \text{ inch-kips}$$

The depth of the foundation
can be found using the
equation:

$$f_c' = \frac{Q}{\frac{1}{2}kdjdb}$$

where Q=bending moment

kd=distance from extreme compressive
fiber to neutral axis (expressed as
a fraction of d)

jd=distance between resultants of the
tensile and compressive forces on the
section(expressed as a fraction of d)

d=distance from extreme fiber to steel
reinforcement

b=width of section

f_c'=maximum compressive stress allowed
in concrete

Assuming that f_c=1000 psi; j=.875, k=.375; then
d=10 inches for Q=1835 inch-kips.

Allowing 4 inches of cover
the depth of the foundation is 14 inches.

Weight of foundation:

Concrete	$150x10x10x\frac{14}{12}$ =	17.5 kips
Reinforcing Steel		2.5 kips
Total		20 kips

<u>Structure Constants</u>

Mass of foundation and lower half of columns:

$$M_1 = \frac{20000+50}{386} = 52 \text{ slugs}$$

Equivalent mass of camera, electronic equipment, floor girders and upper half of columns:

$$M_2 = \frac{200000+3200+50}{386} = 526 \text{ slugs}$$

Vertical stiffness of soil: c_u=7.6 from Table I

$$K_1(\text{vert}) = 7.6 \times 2000 \times 100 = 1520 \text{ kips/inch}$$

Lateral stiffness of soil:

$$K_1(\text{lat}) = \tfrac{1}{2}K_1(\text{vert}) = 760 \text{ kips/inch}$$

Vertical stiffness of columns:

The vertical stiffness of the columns is much greater than the vertical stiffness of the soil mass and the structure will essentially respond as a single degree of freedom system in the vertical direction. For vertical motion assume $K_2/K_1=10$ and $M_2/M_1=50$.

Lateral stiffness of columns:

An expression for the lateral stiffness of column fixed at both ends can be derived using the Moment-Area Theorem.

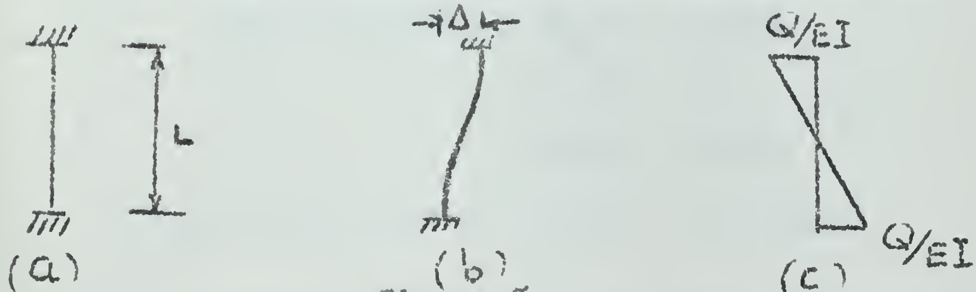

Figure 5

If the top of the column shown in figure 5 (a) is displaced a unit distance as shown in figure 5 (b) the moment diagram shown in figure 5 (c) results. Taking moments about the base of the moment diagram

$$\Delta = \tfrac{1}{2} \times \frac{Q}{EI} \times \frac{L}{2} \times \frac{5}{6}L - \tfrac{1}{2} \times \frac{Q}{EI} \times \frac{L}{2} \times \frac{L}{6} = \frac{QL^2}{6EI}$$

and setting unity and solving for Q

$$Q = \frac{6EI}{L^2}$$

Q is the moment induced at each end of the column when one end is displaced a unit distance as shown in figure 5(b). The shear in the column may be found by adding the moments at each end and dividing by the length of the column. The shear so determined is equivalent to the lateral stiffness of the column.

$$K = \frac{12EI}{L^3}$$

$$K_2 = \frac{4 \times 12 \times 30 \times 10^6 \times 7.23}{36 \times 36 \times 36} = 223 \text{ kips/inch}$$

Mass and stiffness ratios:

Vertical motion $K_2/K_1 = 10$

$M_2/M_1 = 50$

$K_1/M_1 = 29200$

Lateral motion $K_2/K_1 = 0.294$

$M_2/M_1 = 10$

$K_1/M_1 = 14600$

Fundamental natural circular frequency:

Vertical motion $p_1 = 46$ radians/second
from chart 1, appendix B

Lateral motion $p_1 = 15$ radians/second
from chart 2, appendix B

frequency ratio:

The frequency of the ground displacement was given as .5 cycles/second or 3.14 radians/second.

Vertical motion $w/p_1 = 0.682$

Lateral motion $w/p_1 = 0.208$

<u>Response</u>

Displacement:

Vertical motion

Foundation 8.33x1.2=10 inches chart 6 appendix C

Platform 8.33x1.2=10 inches chart 12 appendix C

Lateral motion

Foundation 8.33x1.2=10 inches chart 2, appendix C

Platform 8.33x1.2=10 inches chart 8, appendix C

Acceleration:

From the particular solution of the governing differential equations of motion it is seen that the acceleration of each mass is approximately equal to the frequency of the forcing function squared multiplied by the displacement of the mass.

In both vertical and lateral motion the acceleration of the foundation and platform is

$$(3.14)^2 \times 10 = 98 \text{ inches/second}^2$$

which is less than 0.26g.

Therefore the initial design satisfies the conditions of the problem.

C. DESIGN EXAMPLE NO. 6

A Naval construction Battalion building a concrete airstrip in a remote area requires a vibrating screed to expedite their placing operations and being resourceful they have fabricated a screed from surplus materials. The screed itself is made of wood (density= 50 pounds/ ft^3) 20 feet long by 12 inches high by 10 inches thick. A small gasoline engine and vibrator unit (total weight= 50 pounds) are mounted at the center of the screed. The vibrator unit consists of a 5 pound weight mounted on a 10 inch flywheel and the eccentricity of the 5 pound weight can be varied. The engine turns the flywheel at 500 RPM. Lateral vibrations are transmitted to the screed through a rubber shock mount whose equivalent stiffness is equal to 5200 pounds/inch. Neglecting vertical vibrations and assuming that the maximum lateral amplitude of the screed is limited to 0.1 inch to prevent segregation, determine the eccentricity of the 5 pound weight. Assume modulus of elasticity of wood is 1,500,000 pounds/$inch^2$.

Solution

Equivalent mass of screed (assume .625 of total weight):

$$M_1 = \frac{\frac{20 \times 1 \times 10 \times 50 \times .625}{72}}{386} = 1.35 \text{ slugs}$$

Mass of engine and vibrator:

$$M_2 = \frac{50}{386} = .13 \text{ slugs}$$

Stiffness of screed:

Use deflection formula for deflection at center of a simply supported beam due to concentrated load at center. Load that causes a unit deflection is stiffness of beam.

$$K_1 = \frac{48 \times 1.5 \times 10^6 \times 10^3}{2.4 \times 10^6} = 5200 \text{ pounds/ inch}$$

Mass and stiffness ratios:

$$K_2/K_1 = 1$$

$$M_2/M_1 = .1$$

$$K_1/M_1 = 4000$$

Fundamental natural circular frequency:

$$p_1 = 60 \text{ radians / second}$$

from chart 3, appendix B

Frequency ratio:

$$w = \frac{500}{60} \times 6.28 = 52.3 \text{ radians / second}$$

$$w/p_1 = 0.872$$

Entering chart 16, appendix C obtain a multiplier value of 0.021

Therefore $F = \frac{0.1 \times 4000}{400 \times .021} = 47.6$ pounds

If $F = \bar{M} \times \bar{r} \times w^2$

then $\bar{r} = \dfrac{47.6 \times 386}{5 \times 52.3 \times 52.3} = 1.35$ inches

An eccentricity of 1.35 inches will cause the screed to vibrate with an amplitude of 0.1 inch or less.

PART IV. SUMMARY

The example problems presented in Part III though greatly simplified illustrate the procedures and means that should be used in the design of structures involving and in solving the governing differential equations of motion by the charts provided. The steps in the above procedures are:

Step 1 Obtain a suitable design for static conditions.

Step 2 Calculate the mass and respective stiffness for each part of the system and assign subscripts to these values so that the system is analogos to and therefore can be represented by the model discussed in part I.
Note: Units of the mass and stiffness must be consistent. If the stiffness is represented as pounds/inch the mass must be expressed as pound-second^2/inch

Step 3 Calculate the ratios K_2/K_1, M_2/M_1, and K_1/M_1.

Step 4 Enter Charts 1, 2, or 3 of appendix B and obtain the fundamental natural circular frequency, p_1. If the frequency of the second natural mode is ~~denied~~ desired enter charts 4 or 5 of appendix B.

Step 5 Calculate the ratio of the frequency of the forcing function to the fundamental natural circular frequency of the system, w/p_1.

Step 6 Enter the Charts of Appendix C to obtain the response of the system. Use charts 1 through 12 if the forcing function is applied as base motion or charts 13 through 24 if the forcing function is applied to mass two.

The procedures outlined above establish the a basic approach to designing vibrating structures involving two masses and two degrees of freedom. The examples presented show the diversified nature of such design problems and it is the author's intent to demonstrate the flexibility of the charts that have been provided.

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1 Jacobsen, Lydik S., and Ayre, Robert S.: "Engineering Vibrations", McGraw Hill Book Company, Inc., New York, 1958.

2 Rogers, Grover L.: "Dynamics of Frame Structures", John Wiley and Sons, Inc., New York, 1959.

3 Taylor, Donald W.: "Soil Mechanics", John Wiley and Sons, Inc., 1948.

4 Barkan, D. D.: "Dynamics of Bases and Foundations", McGraw Hill Book Company, Inc., 1962.

5 Blume, John A., Newmark, Nathan M., Corning, Leo H.: "Design of Multistory Reinforced Concrete Buildings for Earthquake Motion", Portland Cement Association, 1961.

6 Harris, Cyril M., Crede, Charles E.: "Shock Vibration Handbook", McGraw Hill Book Company, Inc., 1961.

PROGRAM 1

```
  PROGRAM THESIS
  FORCING FUNCTION APPLIED AS BASE MOTION  (HORIZONTAL)
  DIMENSION A(25), B(25), C(25), D(25),
  READ 1, N1,N2,N3,N4,D(L),A(I),B(J),C(K)
1 FORMAT (---------------)
  PRINT 2
2 FORMAT (1H1,8X,1HP,8X,5HK1/M1,4X,5HK2/K1,11X,5HM2/
     M1,12X,3HW/P,13X
 1,2HX1,14X,2HX2,9X,16HREL DISPLACEMENT)
  DO 500 L=1,N1
  DO 500 I=1,N2
  DO 500 J=1,N3
  DO 500 K=1,N4
  RATIO=D(L)
  STIF=A(I) * D(L)
  SLUG=B(J)
  WBYP=C(K)
  CALL ALPHA (RATIO,STIF,SLUG,ALPHA1,ALPHA2,P,P2)
  W=P*WBYP
  CALL BIGX(RATIO,STIF,SLUG,W,BIGX1,BIGX2)
  X1=ABSF((W/P)*((BIGX2-ALPHA2*BIGX1)/(ALPHA1-ALPHA2)))
 1+ABSF((W/P2)*((BIGX2-ALPHA1*BIGX1)/(ALPHA1-ALPHA2)))
 2+ABSF(BIGX1)
  X2=ABSF(ALPHA1*(W/P)*((BIGX2-ALPHA2*BIGX1)/(ALPHA1-ALPHA2)))
 1+ABSF(ALPHA2*(W/P2)*((BIGX2-ALPHA1*BIGX1)/(ALPHA1-ALPHA2)))
```

PROGRAM 1 (Con't)

```
     2+ABSF(BICX1)
      X2=ABSF(ALPHA1*(W/P)*((BIGX2-ALPHA2*BIGX1)/(ALPHA1-ALPHA2))
     1+ABSF(ALPHA2*(W/P2)*((BIGX2-ALPHA1*BIGX1)/(ALPHA1-ALPHA 2)))
     2+ABSF(BIGX2)
      RELX=X2-X1
  500 PRINT 3,P,RATIO,A(I),SLUG,WBYP,X1,X2,RELX
    3 FORMAT(F8.2,F16.2,F8.2,2F16.2,3F18.2)
      END
      THE FOLLOWING SUBROUTINE GENERATES AMPLITUDE RATIOS
      SUBROUTINE ALPHA (A,B,C,D,E,G,H)
      G=SQRTF(((A+B)/2.+B/(2.*C))-SQRTF(((A+B)/2.+B/
          (2.*C))**2-A*B/C))
      H=SQRTF(G*G+2.*SQRTF((((A+B)/2.)+B/(2.*C))**2-A*B/C))
      D=(A+B-G*G)/B
      E=(A+B-H*H)/B
      RETURN
      END
      THE FOLLOWING SUBROUTINE GENERATES THE MAXIMUM
          TRANSIENT DISPLACEMENTS
      SUBROUTINE BIGX(A,B,C,U,V,Z)
      V=A*(B/C-U*U)/(U**4-U*U*(A+B+B/C)+A*B/C)
      Z=A*B/C/(U**4-U*U*(A+B+(B/C))+A*B/C)
      RETURN
      END
      END
      FOLLOWING PROGRAM IS BLANK CARD THEN DATA CARDS
```

Appendix A

PROGRAM 2

```
  PROGRAM THESIS
  FORCING FUNCTION APPLIED TO SECOND FLOOR  (HORIZONTAL)
  DIMENSION A(25),B(25),C(25),D(25)
  READ 1,N1,N2,N3,N4,D(L),A(I),B(J),C(K)
1 FORMAT (----------------)
  PRINT 2
2 FORMAT (1H1,6X,1HP,8X,5HK1/M1,4X,5HK2/K1,11X,
      5HM2/M1,12X,3HW/P,13X
 1,2HX1,14X,2HX2,9X,16HREL DISPLACEMENT)
  DO 500 L=1,N1
  DO 500 I=1,N2
  DO 500 J=1,N3
  DO 500 K=1,N4
  RATIO=D(L)
  STIF=A(I) * D(L)
  SLUG=B(J)
  WBYP=C(K)
  CALL ALPHA(RATIO,STIF,SLUG,ALPHA1,ALPHA2,P,P2)
  W=P*WBYP
  CALL BIGX(RATIO,STIF,SLUG,W,BIGX1,BIGX2)
  X1=ABSF((W/P)*((BIGX2-ALPHA2*BIGX1)/(ALPHA1-ALPHA2)))
 1+ABSF((W/P2)*((BIGX2-ALPHA1*BIGX1)/(ALPHA1-ALPHA2)))
 2+ABSF(BIGX1)
 X2=ABSF(ALPHA1*(W/P)*((BIGX2-ALPHA2*BIGX1)/(ALPHA1-ALPHA2)))
1+ABSF(ALPHA2*(W/P2)*((BIGX2-ALPHA1*BIGX1)/(ALPHA1-ALPHA2)))
2+ABSF(BIGX2)
```

PROGRAM 2 (Con't)

```
    RELX=X2-X1
500 PRINT 3,P2,RATIO,A(I),SLUG,WBXP,X1,X2,RELX
  3 FORMAT (F8.2,F16.2,F8.2,2F16.2,3F18.2)
    END
    THE FOLLOWING SUBROUTINE GENERATES AMPLITUDE RATIOS
    SUBROUTINE ALPHA(A,B,C,D,E,G,H)
    G=SQRTF(((A+B)/2.+B/(2.*C))-SQRTF(((A+B)/2.+B/(2.*C))
        **2-A*B/C))
    H=SQRTF(G*G+2.*SQRTF((((A+B)/2.)+B/(2.*C))**2-A*B/C))
    D=(A+B-G*G)/B
    E=(A+B-H*H)/B
    RETURN
    END
    THE FOLLOWING SUBROUTINE GENERATES THE MAXIMUM
        TRANSIENT DISPLACEMENTS
    SUBROUTINE DISX(A,B,C,U,V,Z)
    V=10000.*B/C/(U**4-U*U*(A+B+(B/C))+A*B/C)
    Z=10000.*(B/C-U*U)/(U**4-U*U*(A+B+B/C)+A*B/C)
    RETURN
    END
    END
    FOLLOWING PROGRAM IS BLANK CARD THEN DATA CARDS
```

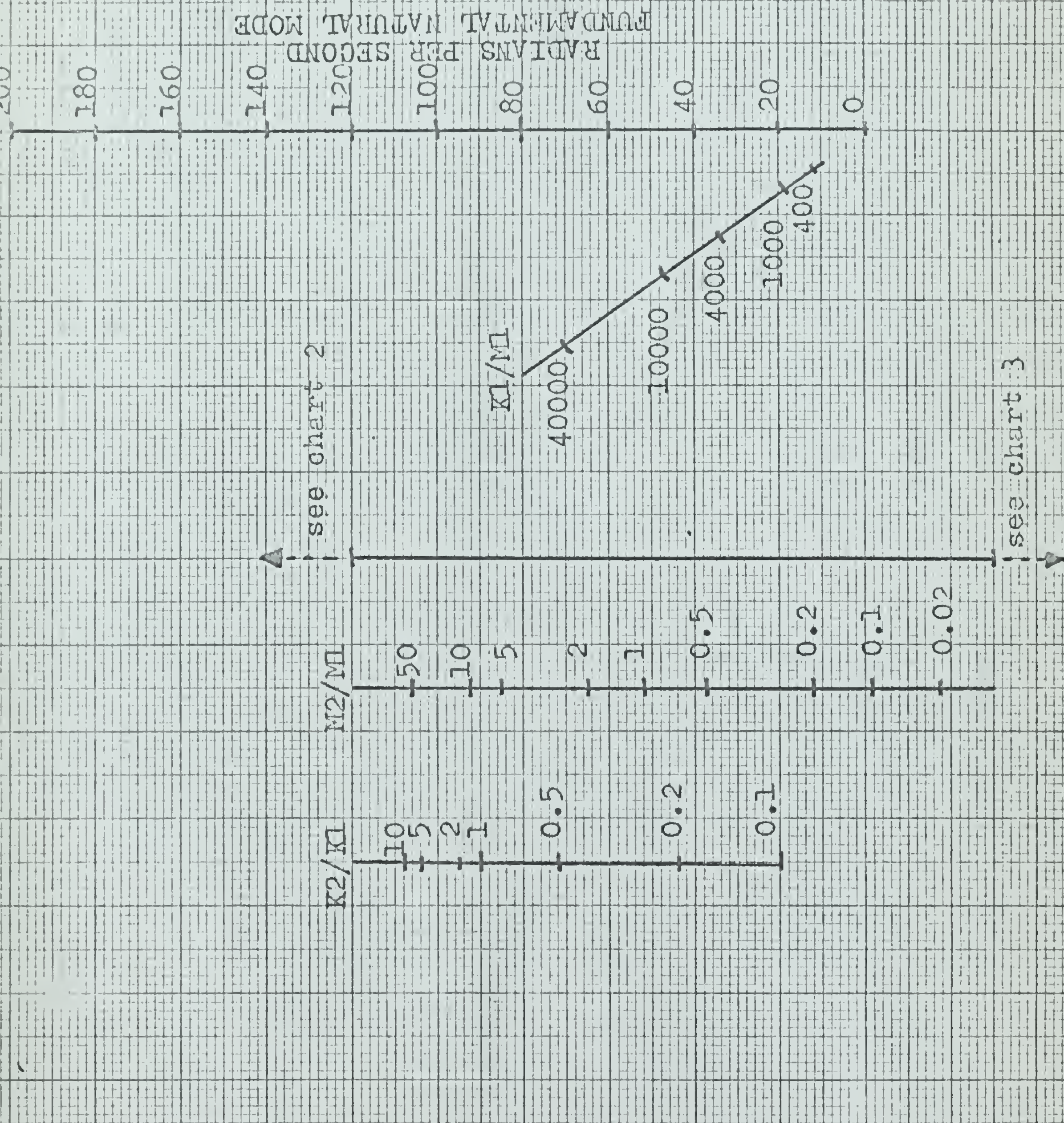

FUNDAMENTAL NATURAL MODE
RADIANS PER SECOND
200 180 160 140 120 100 80 60 40 20 0
K1/M1
40000 10000 4000 1000 400
see chart 2
see chart 3
M2/M1
50 10 5 2 1 0.5 0.2 0.1 0.02
K2/K1
10 5 2 1 0.5 0.2 0.1

CHART 1

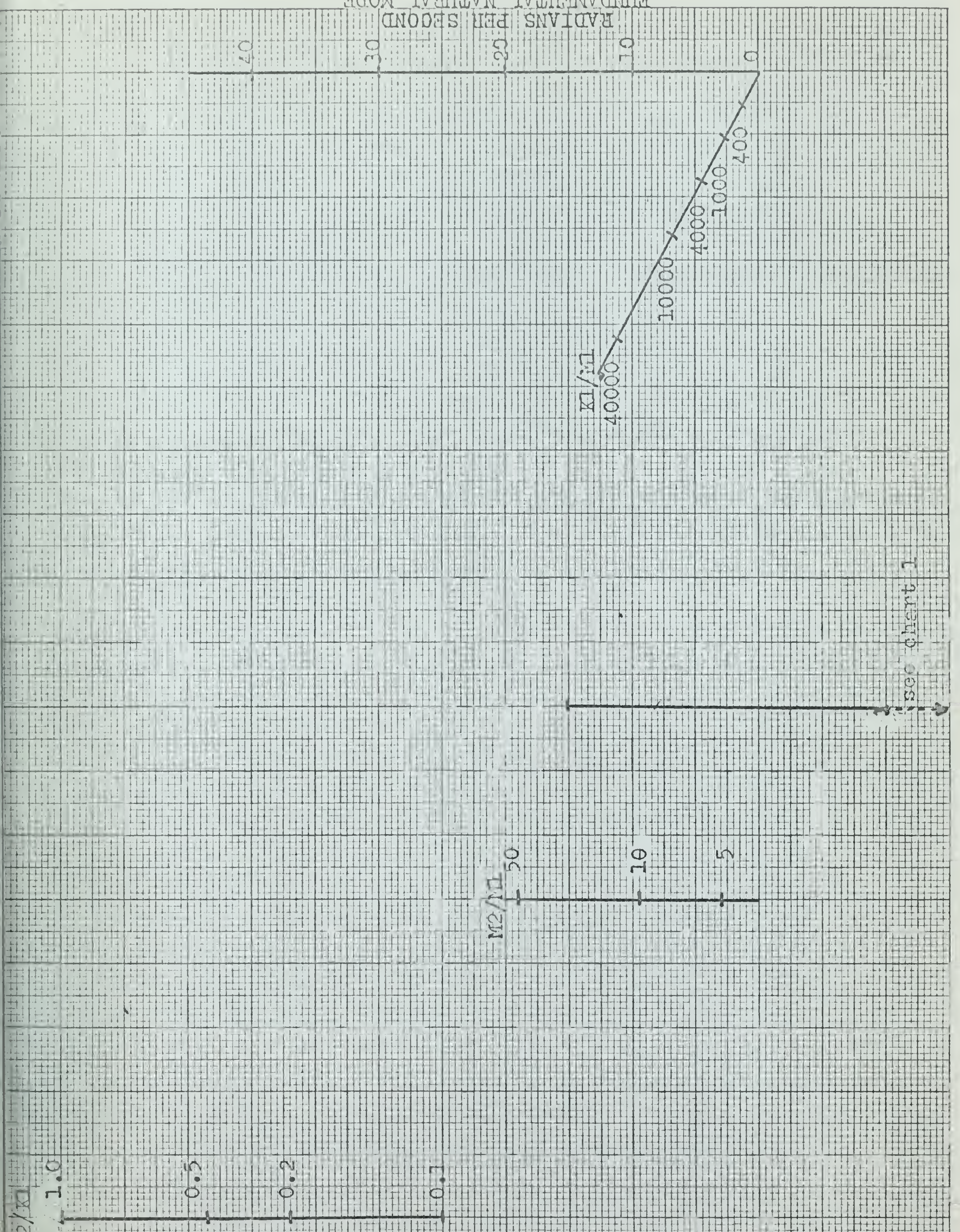

FUNDAMENTAL NATURAL MODE
RADIANS PER SECOND
40
30
20
10
0
K1/M1
40000
10000
4000
1000
400
See chart 1
M2/M1
50
10
5
K2/K1
1.0
0.5
0.2
0.1

CHART 2

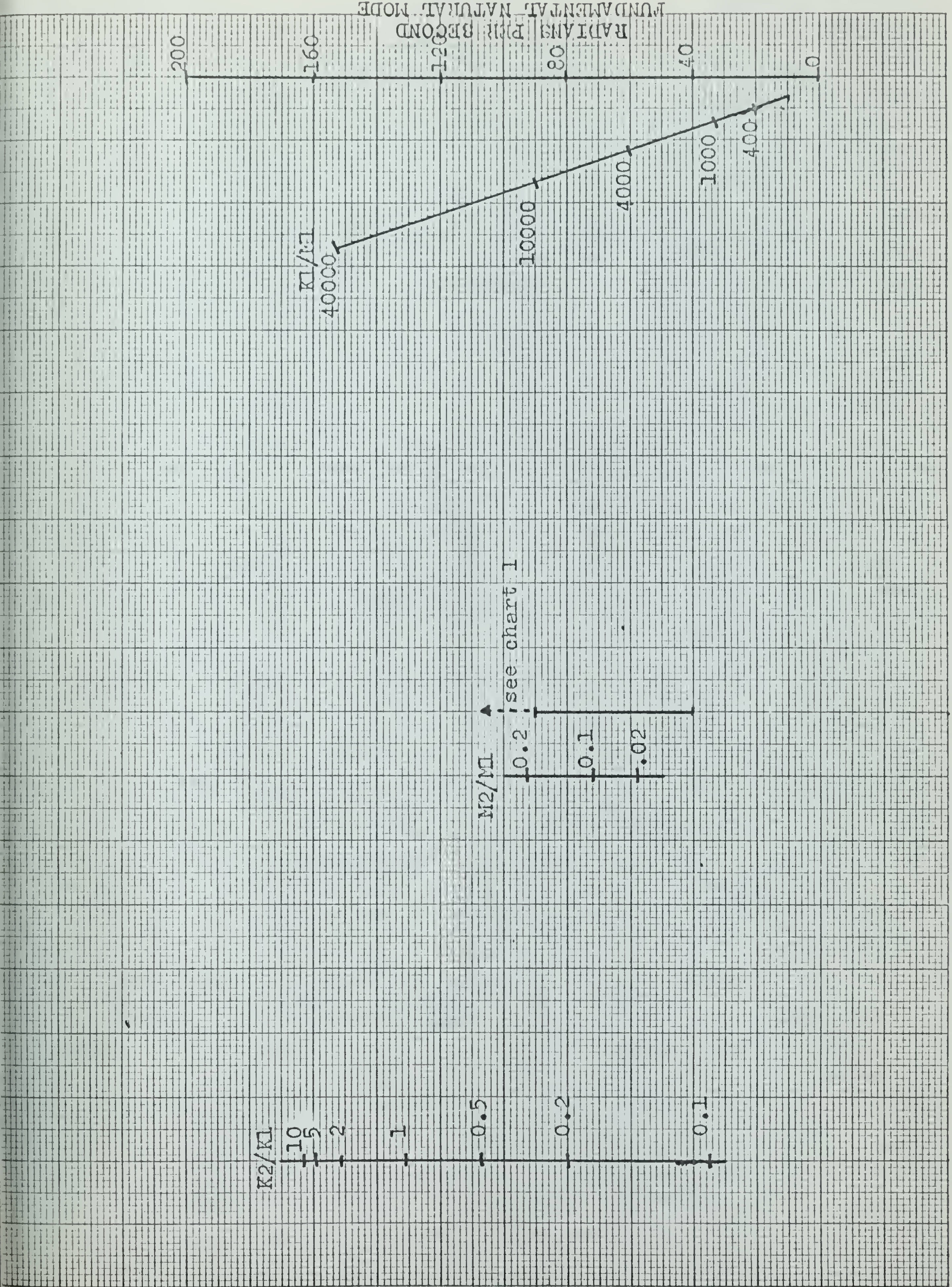
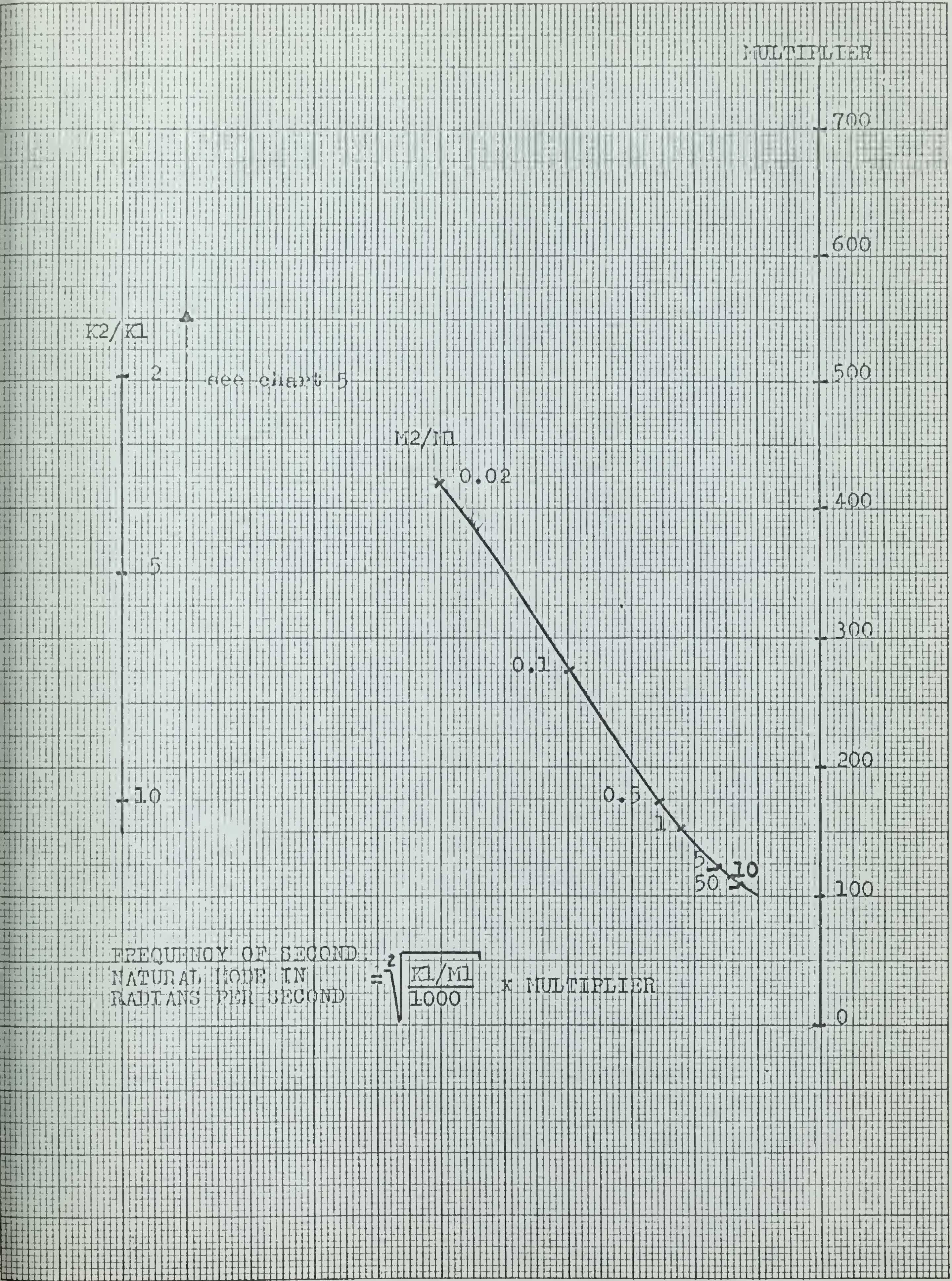

FUNDAMENTAL NATURAL MODE
RADIANS PER SECOND
200
160
120
80
40
0
K1/M1
40000
10000
4000
1000
400
M2/M1
0.2
0.1
.02
see chart 1
K2/K1
10
5
2
1
0.5
0.2
0.1

CHART 3


MULTIPLIER
700
600
500
400
300
200
100
0
K2/K1
2
5
10
see chart 5
M2/M1
0.02
0.1
0.5
1
5
10
50
FREQUENCY OF SECOND NATURAL MODE IN RADIANS PER SECOND $= \sqrt[2]{\frac{K1/M1}{1000}}$ x MULTIPLIER

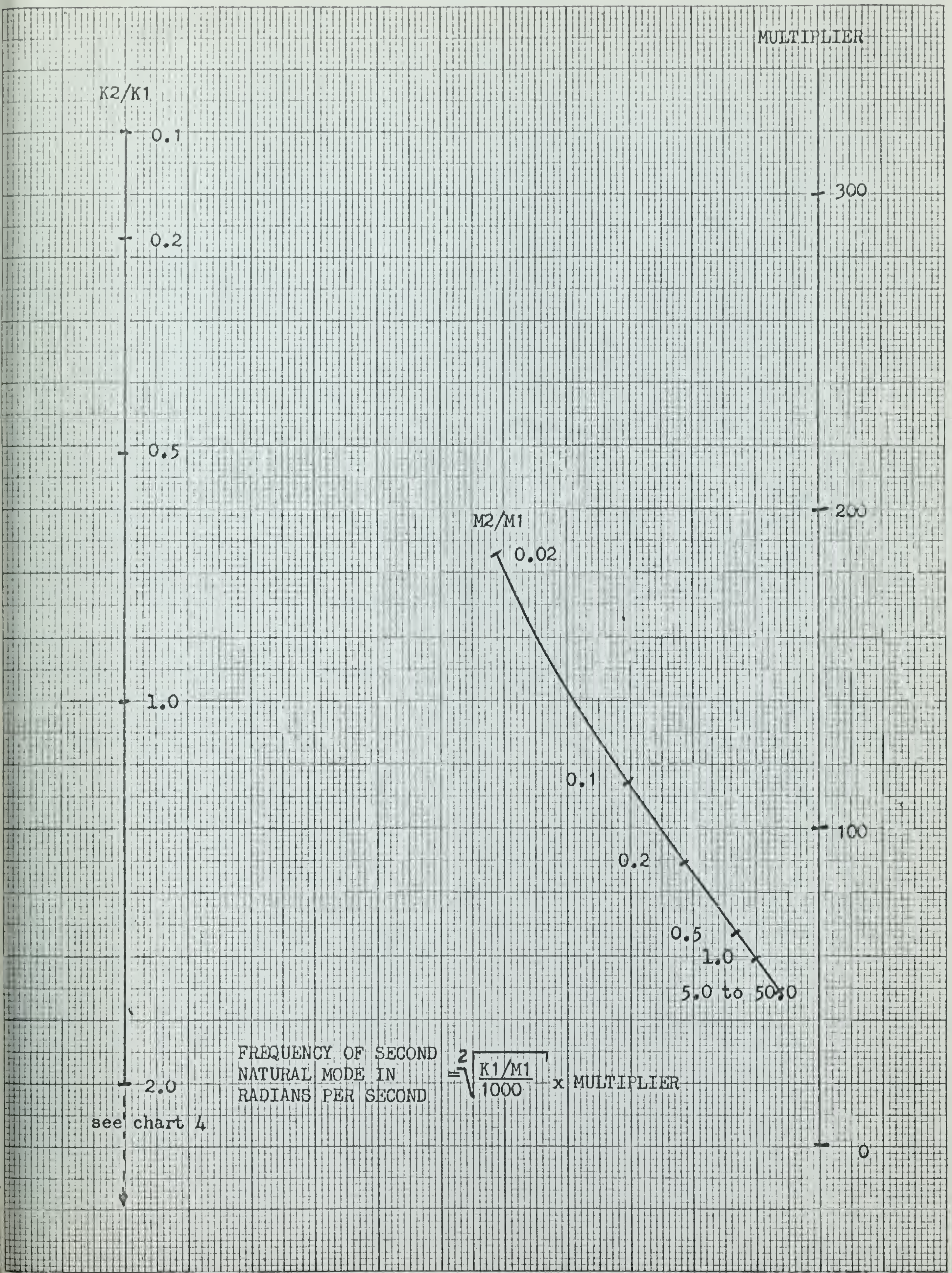


CHART 5 Appendix B

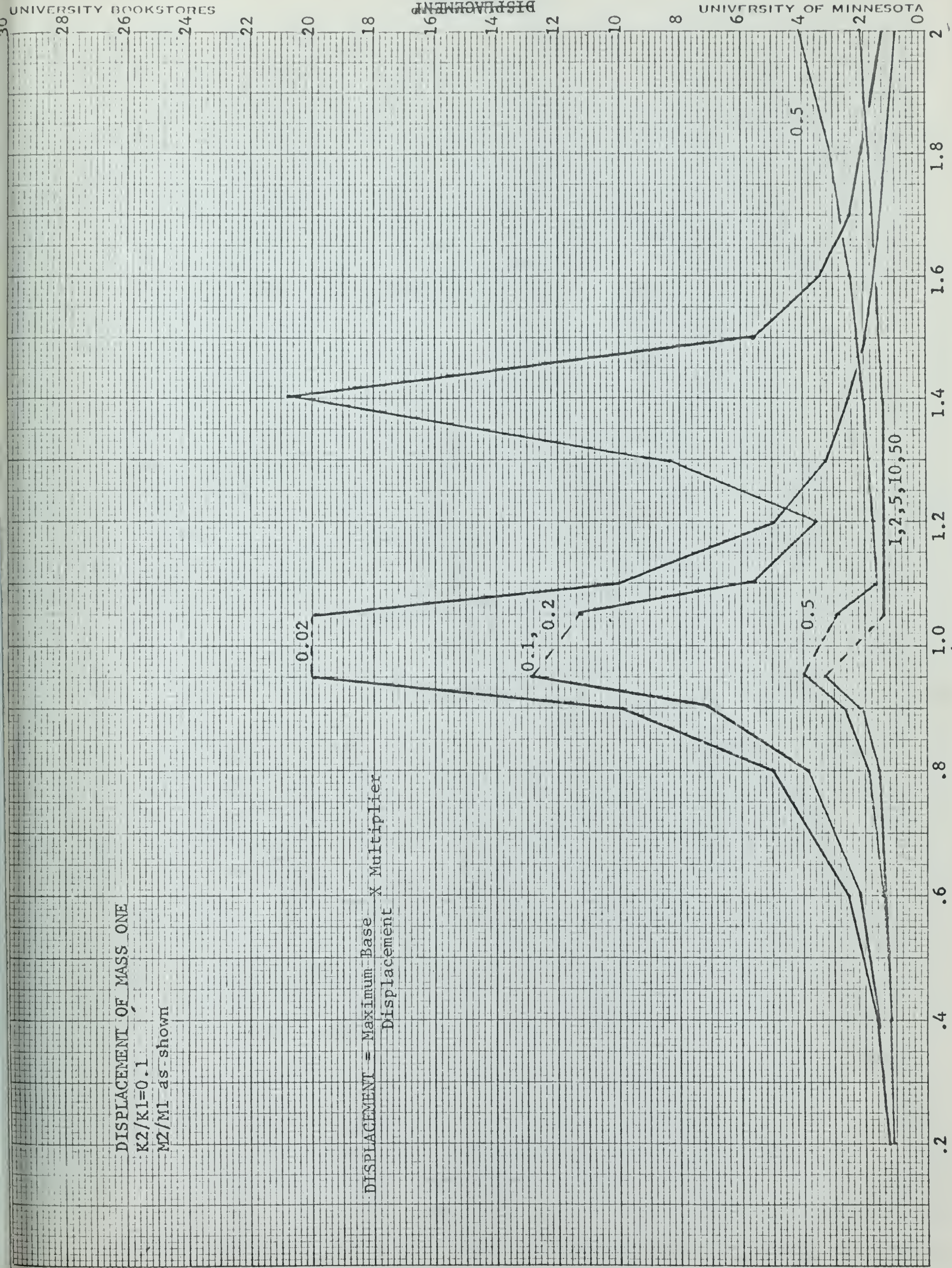


Chart 1

Appendix C

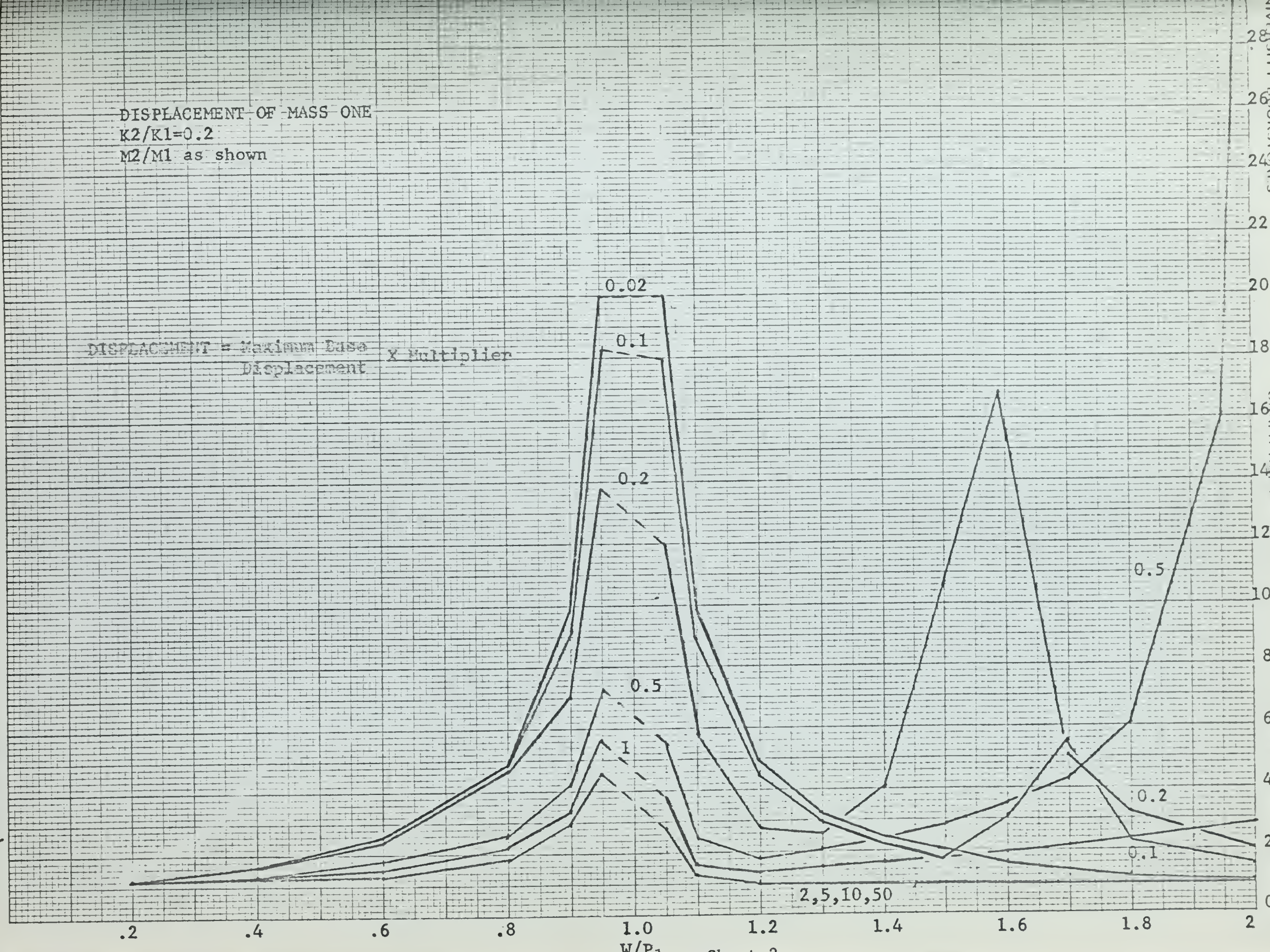


Chart 2

Appendix C

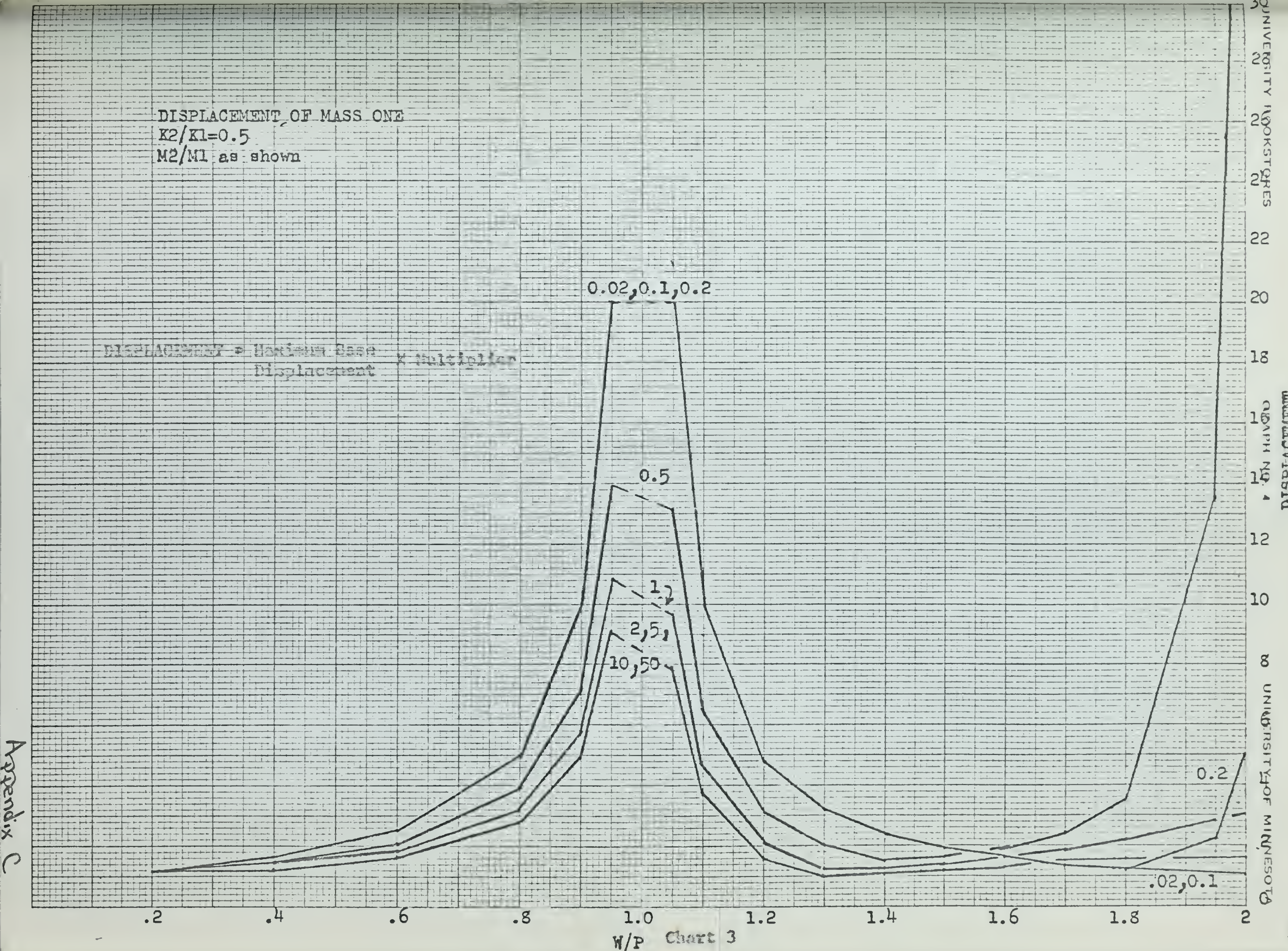
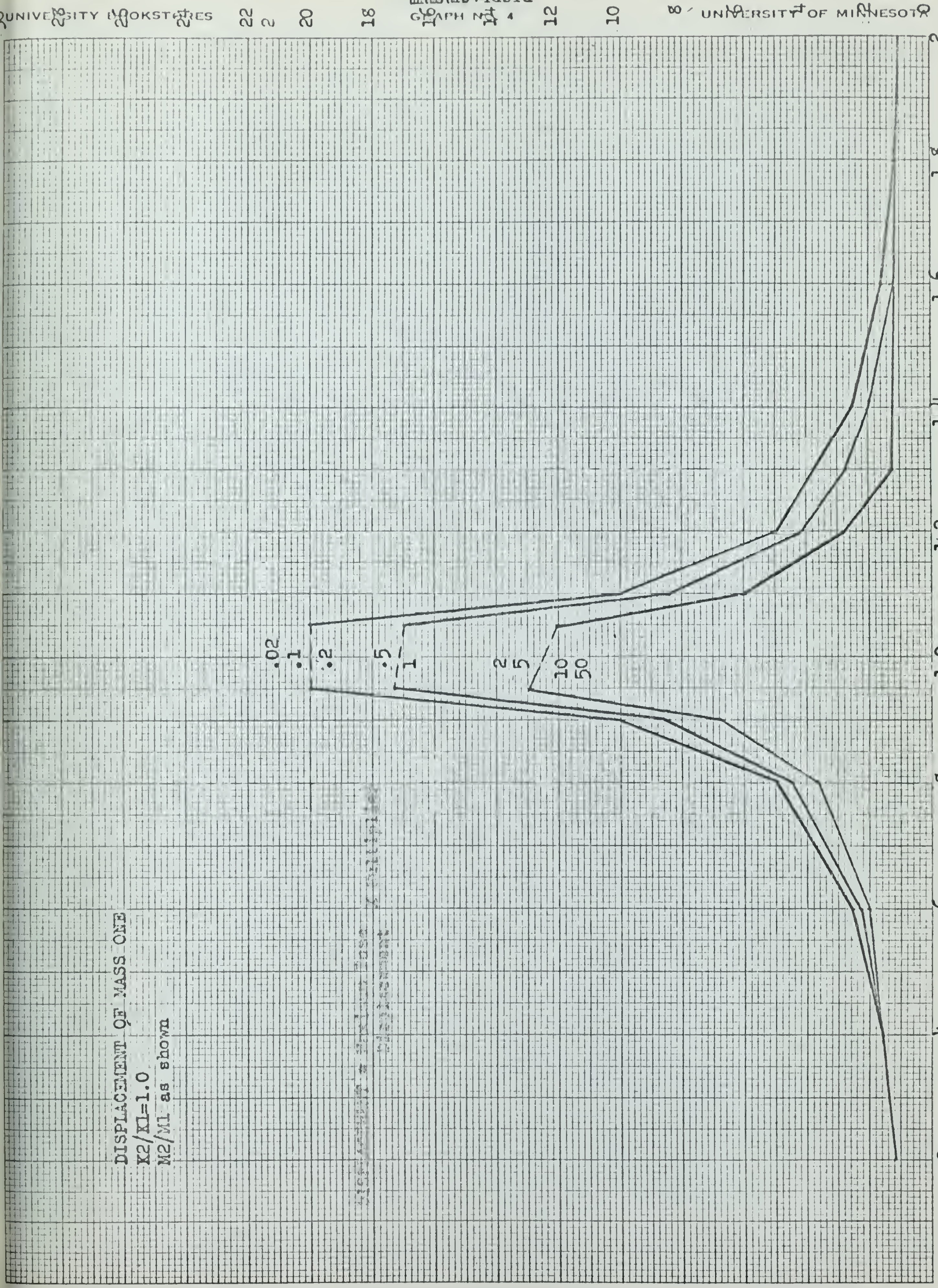
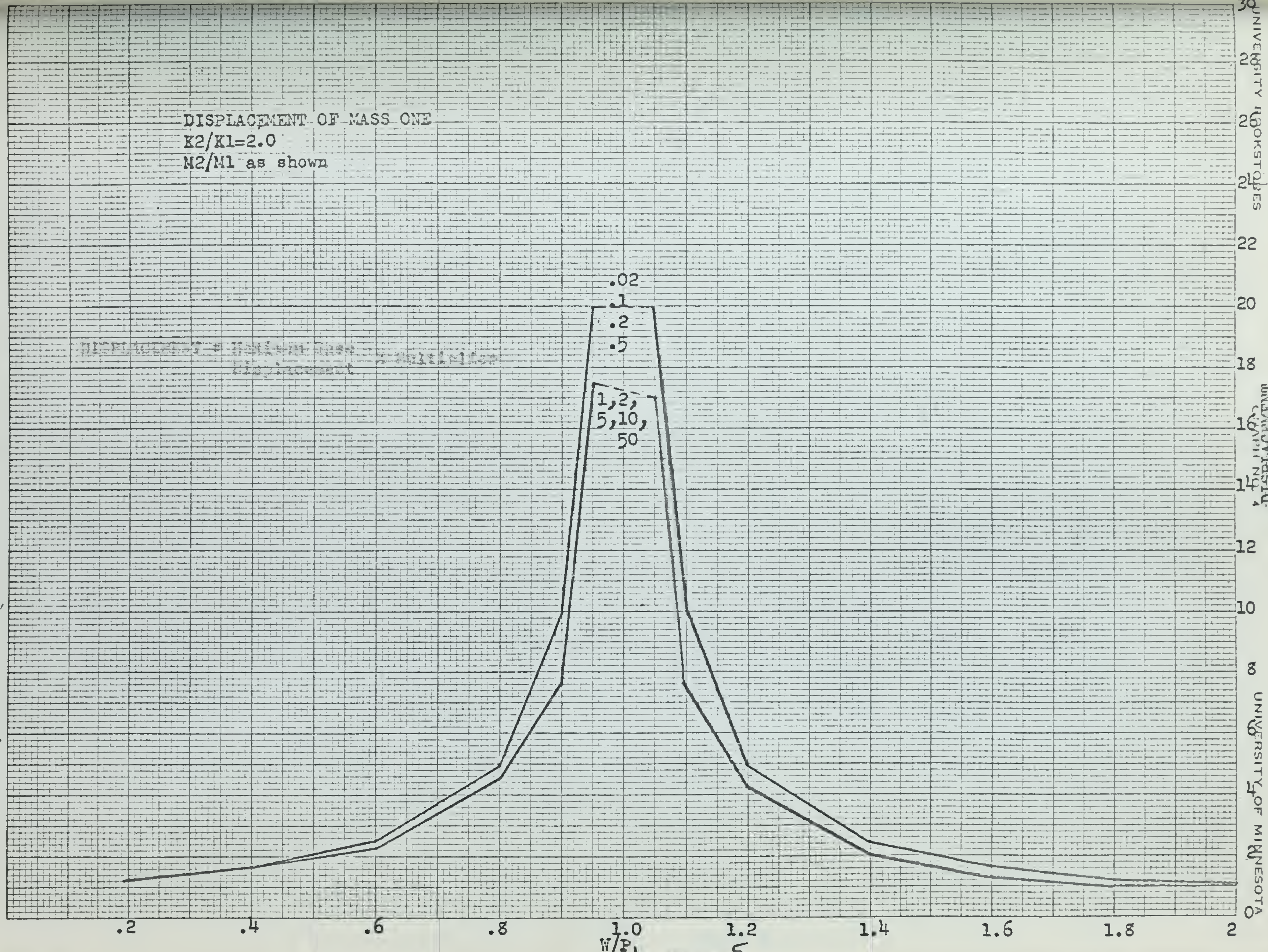


Chart 3

Appendix C

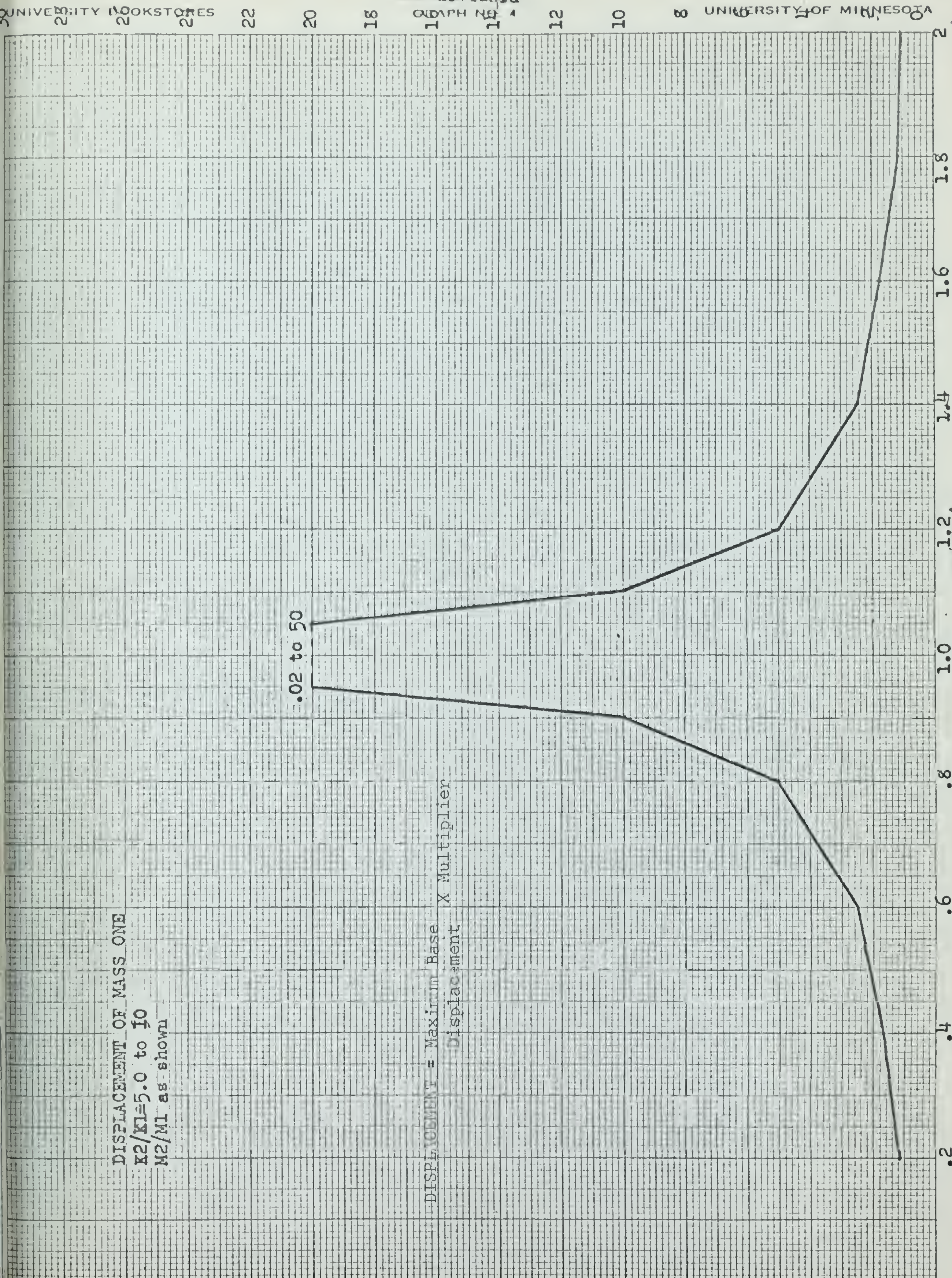

DISPLACEMENT OF MASS ONE
K2/K1=1.0
M2/M1 as shown
DISPLACEMENT MULTIPLIER
W/P1
Chart 4
.02
.1
.2
.5
1
2
5
10
50
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2
8
10
12
14
16
18
20
22
GRAPH No. 4
UNIVERSITY OF MINNESOTA
UNIVERSITY BOOKSTORES

Appendix C



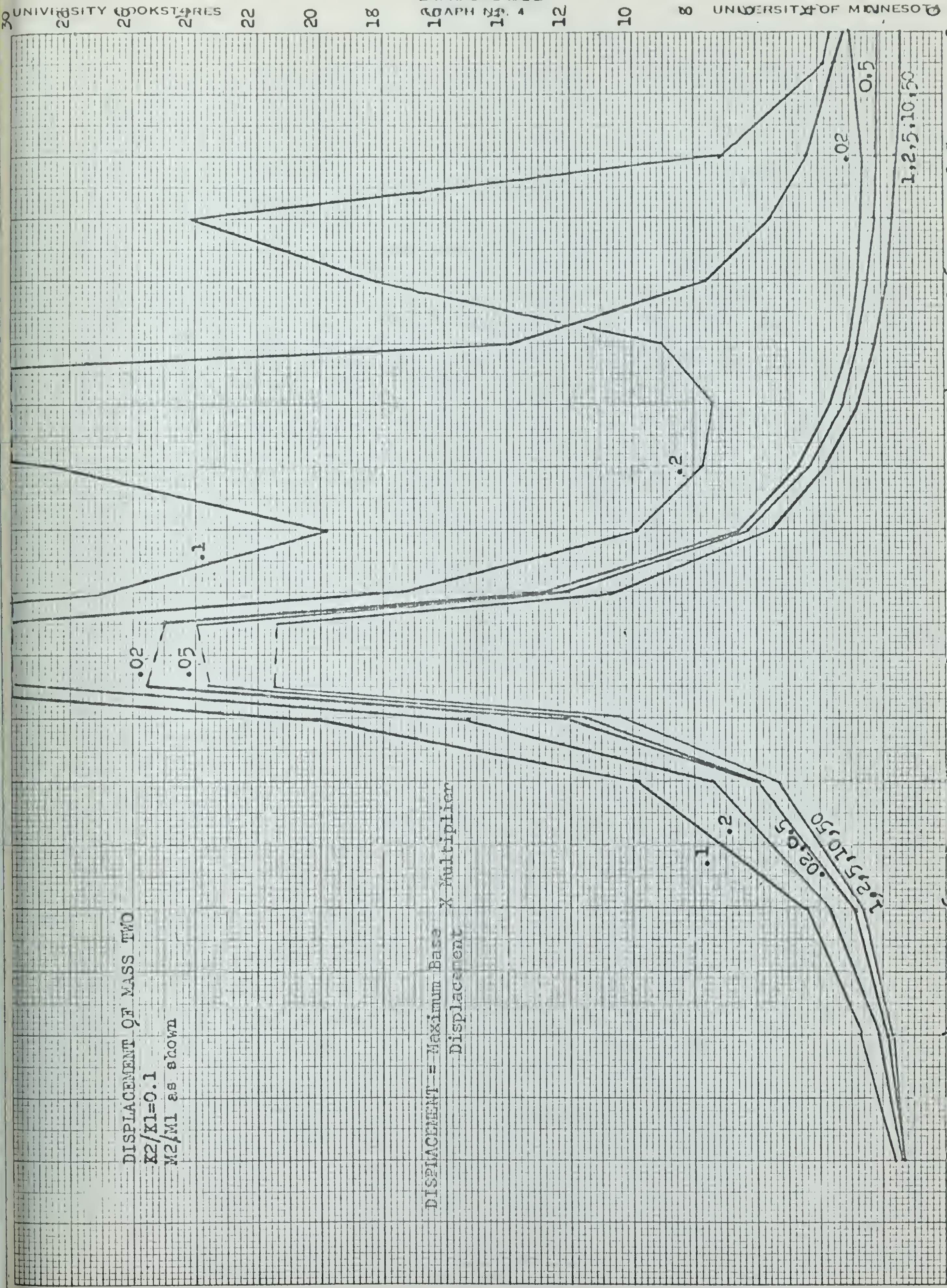
Sheet 5

Appendix C

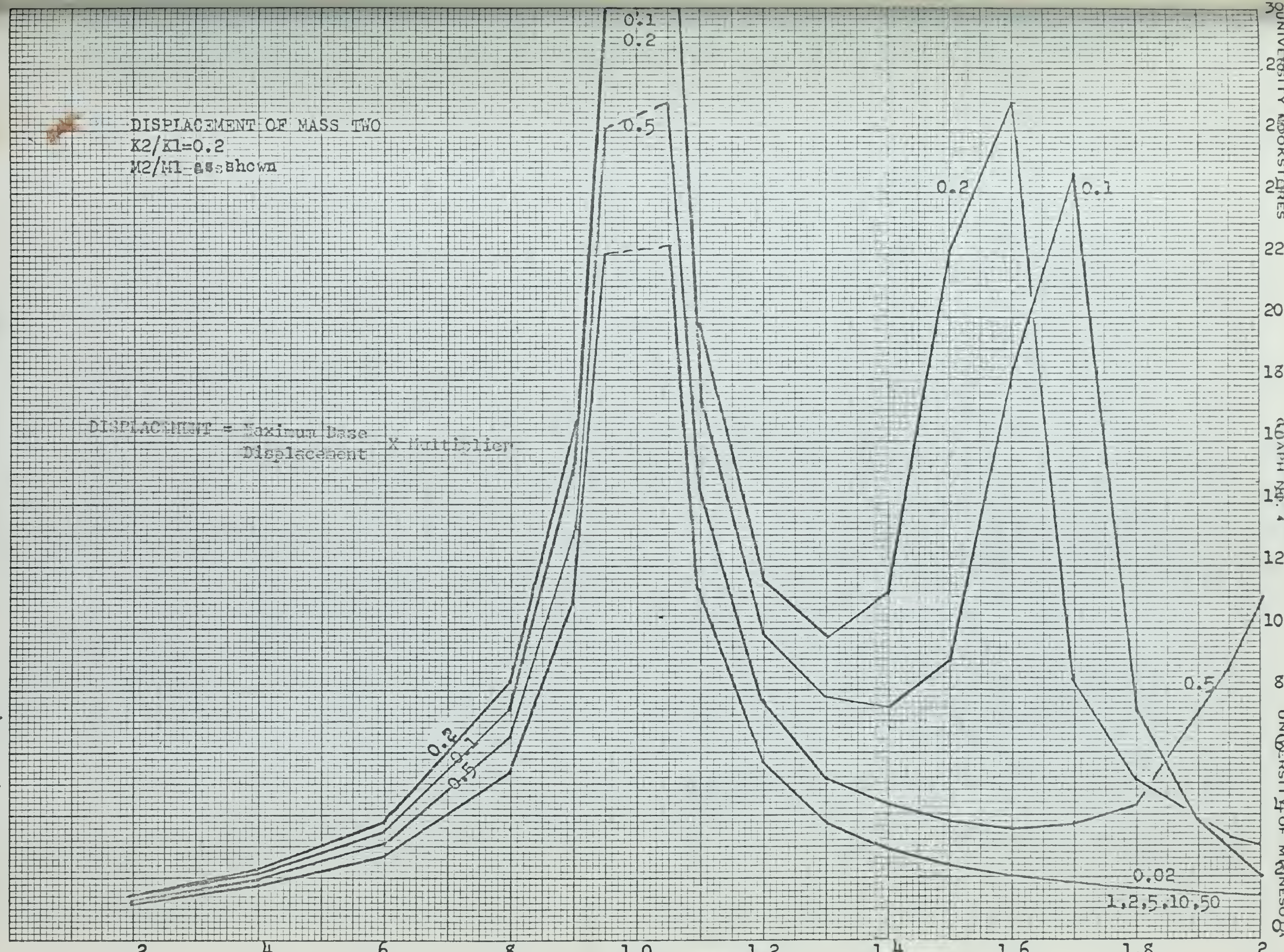


DISPLACEMENT OF MASS ONE
K2/K1=5.0 to 10
M2/M1 as shown
DISPLACEMENT = Maximum Base Displacement X Multiplier
.02 to 50
W/P 1
Chart 6
DISPLACEMENT MULTIPLIER

Appendix C

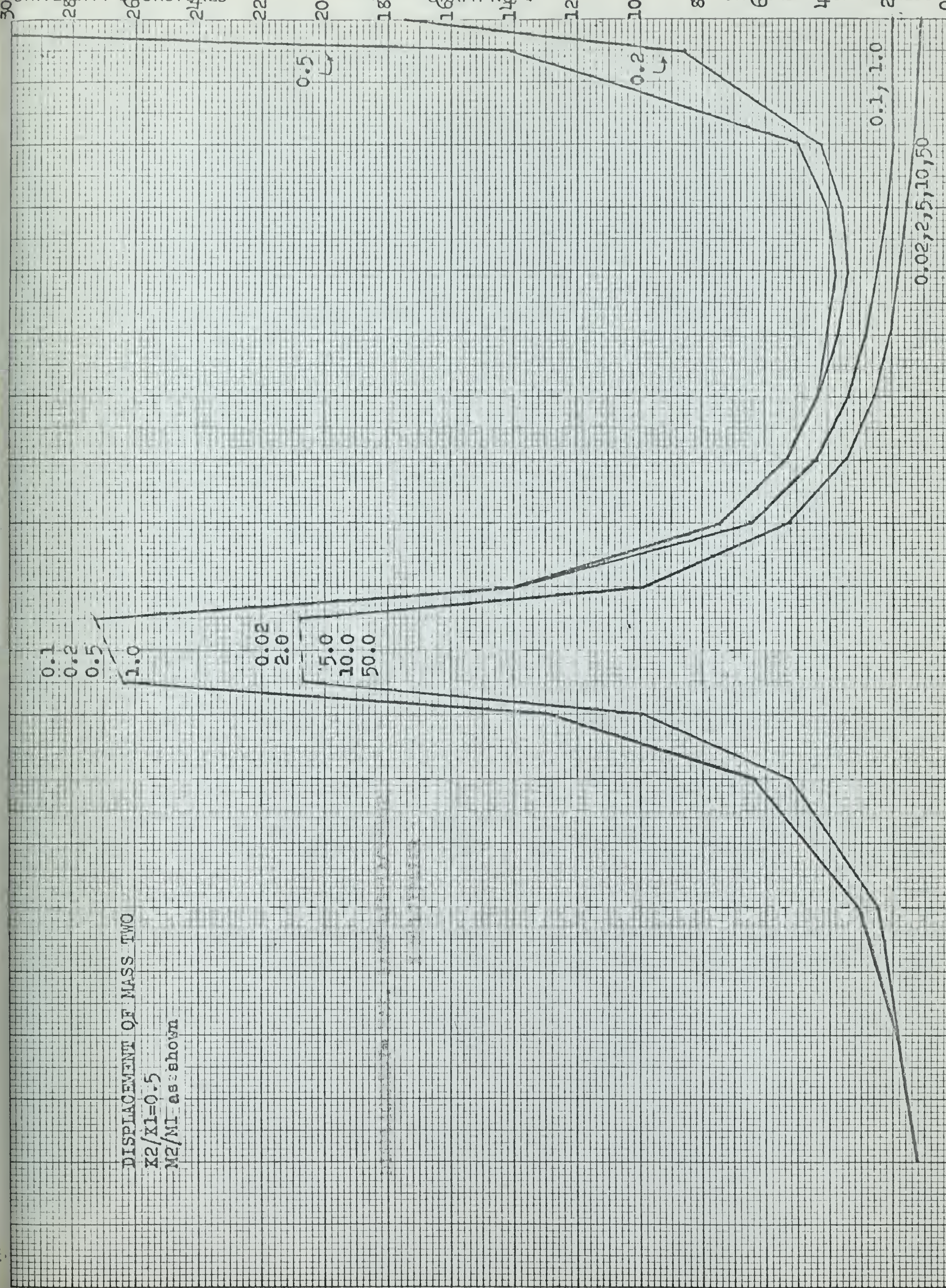

UNIVERSITY BOOKSTORES
GRAPH NO. 4
UNIVERSITY OF MINNESOTA
DISPLACEMENT
MULTIPLIER
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
DISPLACEMENT OF MASS TWO
K2/K1=0.1
M2/M1 as shown
DISPLACEMENT = Maximum Base Displacement X Multiplier
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2
W/P1
Chart 7
.1
.2
.02,0.5
1,2,5,10,50
.02
.05
0.5
1,2,5,10,50

Appendix C

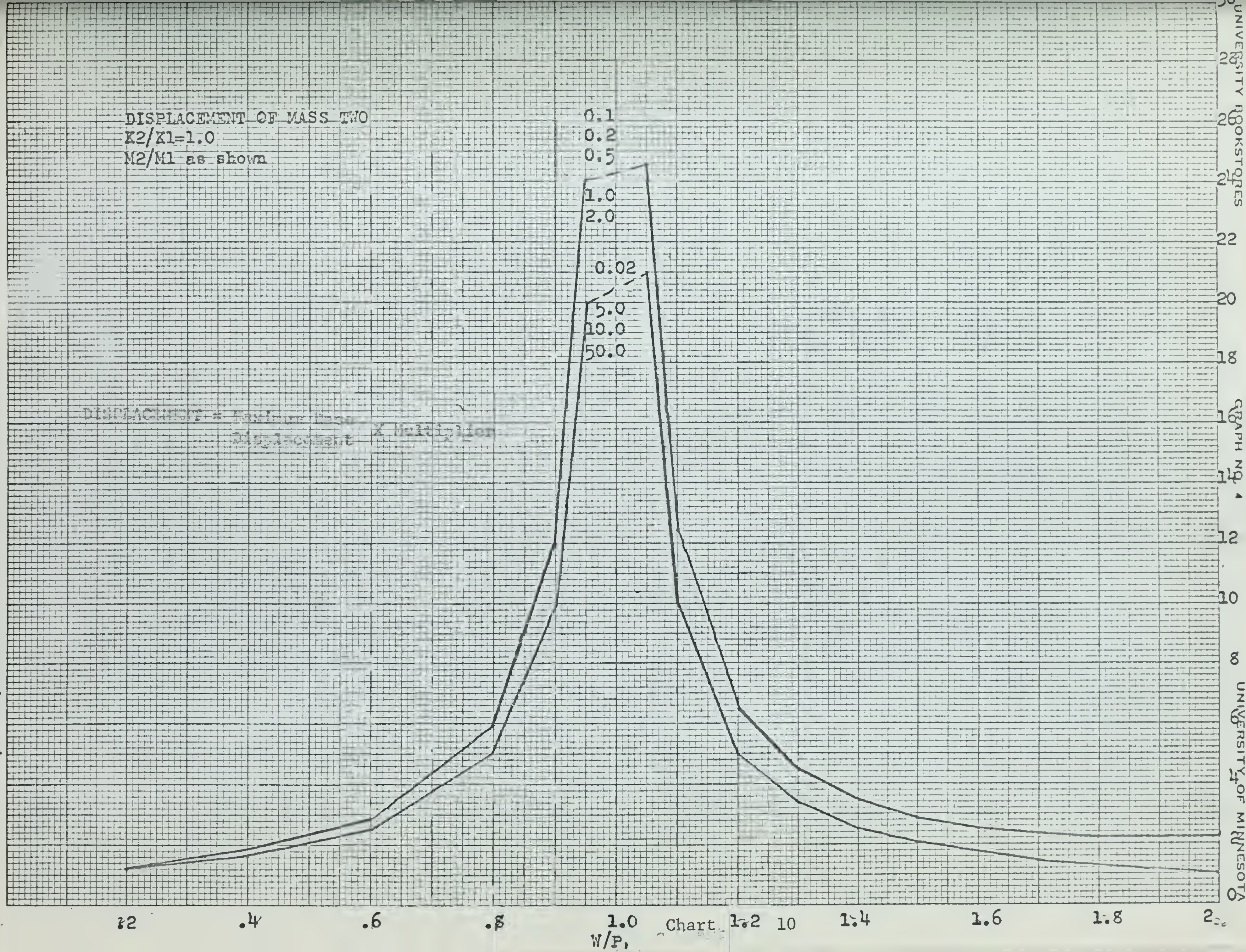


DISPLACEMENT OF MASS TWO
K2/K1=0.2
M2/M1 as shown
DISPLACEMENT = Maximum Base Displacement X Multiplier
0.1
0.2
0.5
0.02
1,2,5,10,50
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2
W/P1
8
10
12
18
20
22
DISPLACEMENT MULTIPLIER

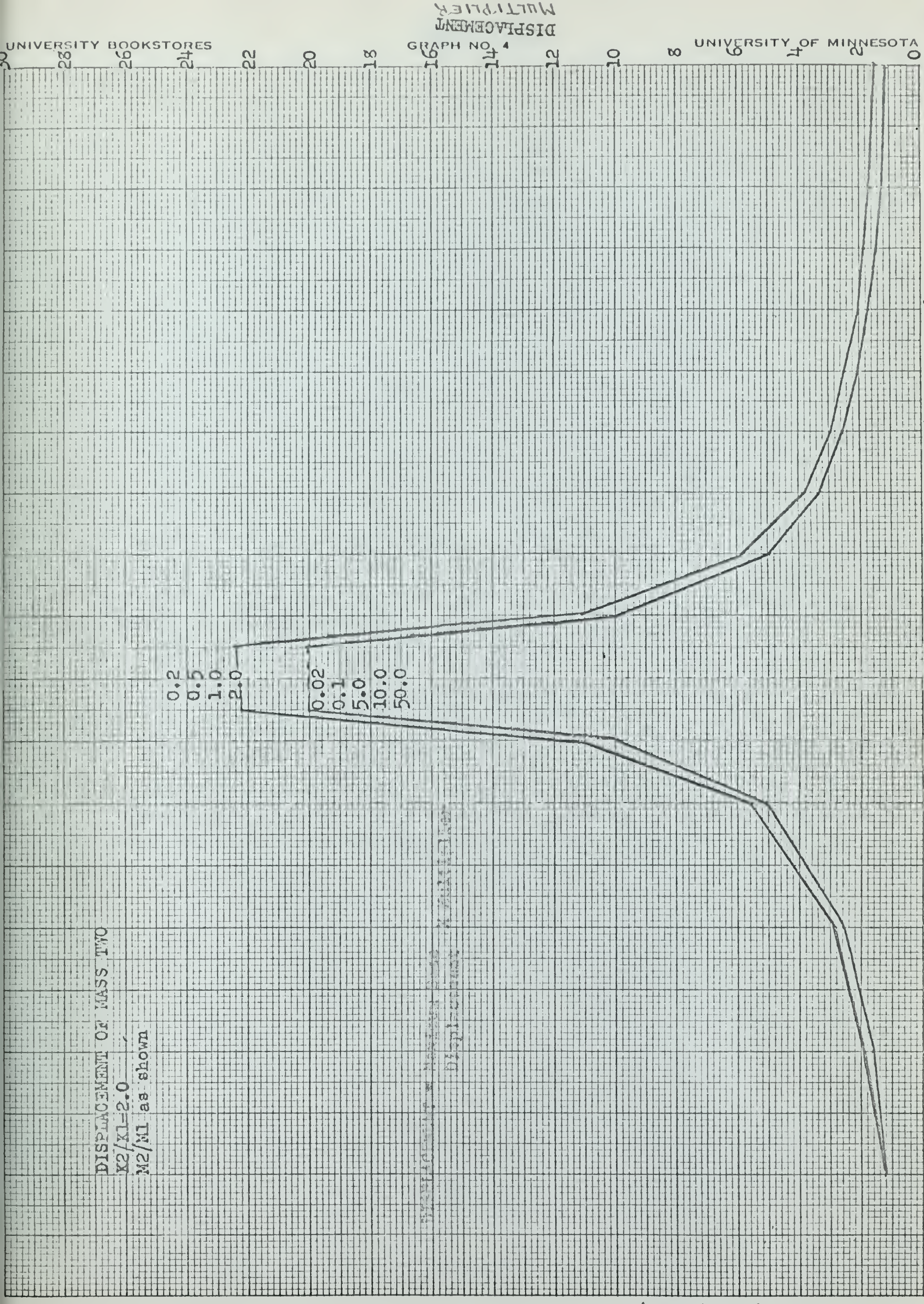
Appendix C


UNIVERSITY BOOKSTORES
GRAPH NO. 4
UNIVERSITY OF MINNESOTA
DISPLACEMENT MULTIPLIER
DISPLACEMENT OF MASS TWO
K2/K1=0.5
M2/M1 as shown
W/P
Chart 9
0.1
0.2
0.5
1.0
0.02
2.0
5.0
10.0
50.0
0.5
0.2
0.1, 1.0
0.02,2,5,10,50

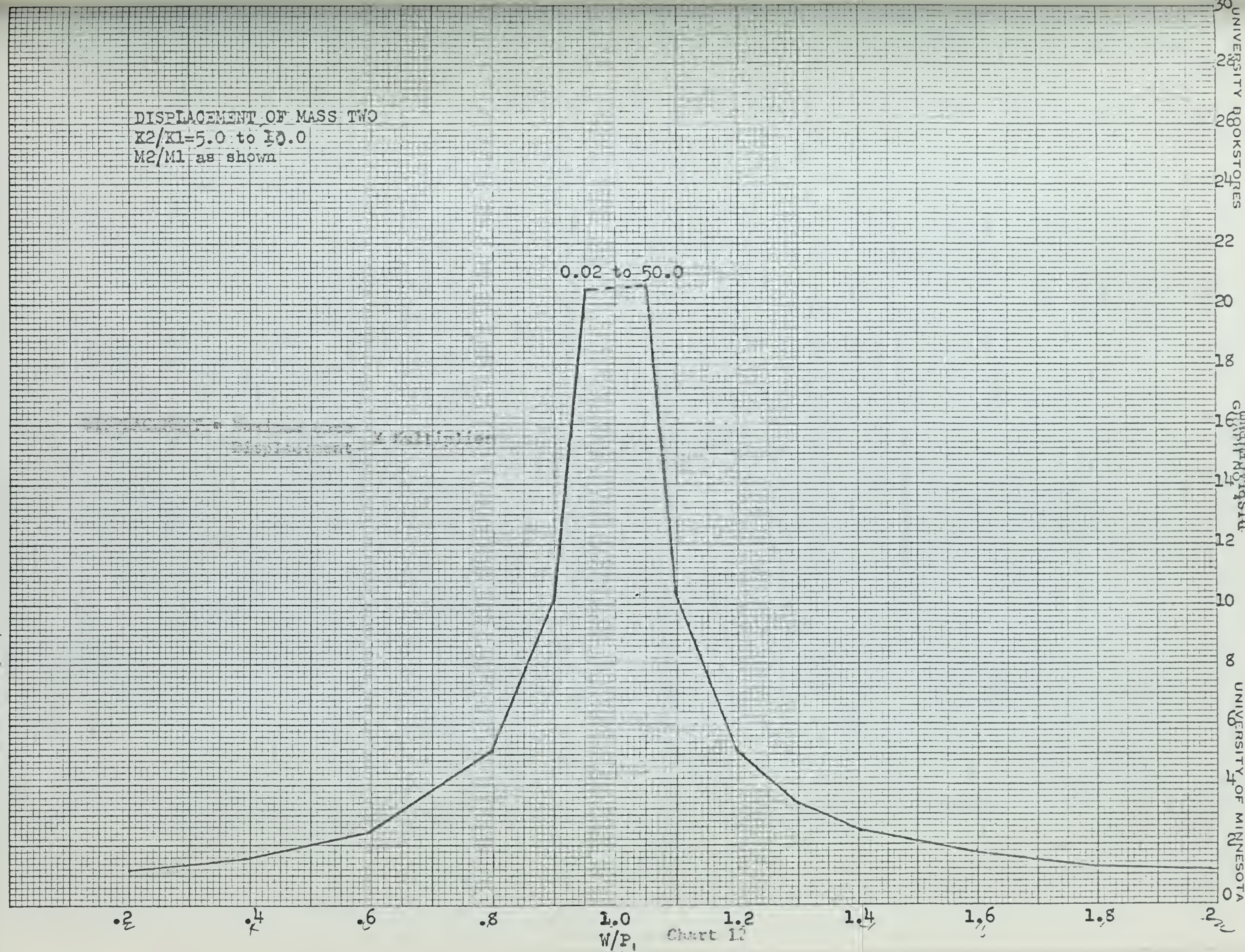
Appendix C


UNIVERSITY BOOKSTORES
GRAPH NO. 4
UNIVERSITY OF MINNESOTA
DISPLACEMENT OF MASS TWO
K2/K1=1.0
M2/M1 as shown
0.1
0.2
0.5
1.0
2.0
0.02
5.0
10.0
50.0
DISPLACEMENT = Maximum Mass Displacement X Multiplier
DISPLACEMENT MULTIPLIER
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
W/P, Chart 10

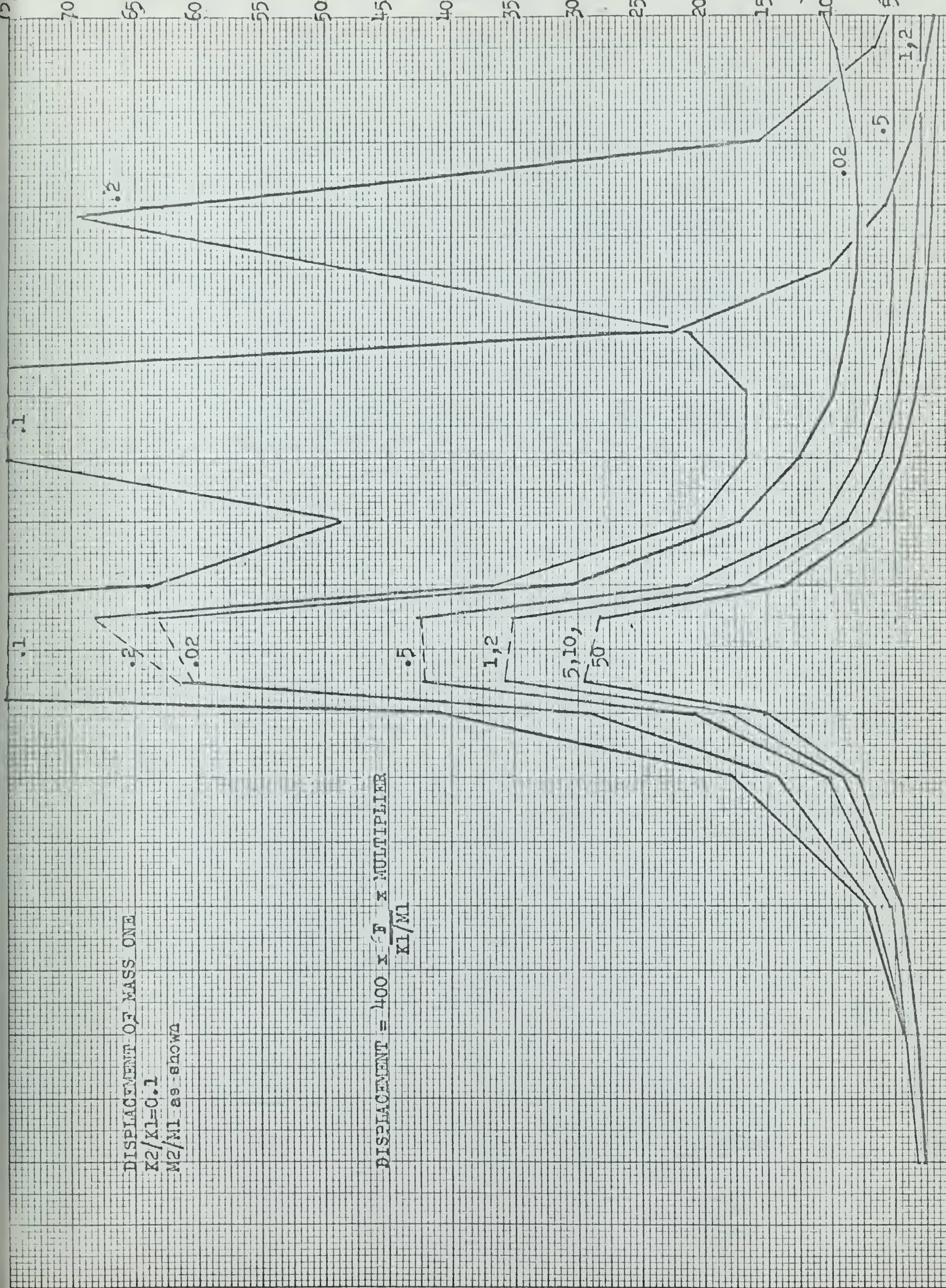
Appendix C


UNIVERSITY BOOKSTORES
GRAPH NO. 4
UNIVERSITY OF MINNESOTA
DISPLACEMENT MULTIPLIER
DISPLACEMENT OF MASS TWO
K2/K1=2.0
M2/M1 as shown
0.2
0.5
1.0
2.0
0.02
0.1
5.0
10.0
50.0
W/P1
Chart 11

Appendix C


DISPLACEMENT OF MASS TWO
K2/K1=5.0 to 10.0
M2/M1 as shown
0.02 to 50.0
DISPLACEMENT MULTIPLIER
W/P1
Chart 12
UNIVERSITY BOOKSTORES
UNIVERSITY OF MINNESOTA

Appendix C


UNIVERSITY BOOKSTORES
GRAPH NO. 4
UNIVERSITY OF MINNESOTA
MULTIPLIER × 10-3
DISPLACEMENT OF MASS ONE
K2/K1=0.1
M2/M1 as shown
DISPLACEMENT = 400 x F/(K1/M1) x MULTIPLIER
W/P1
Chart 13
.1
.2
.02
.5
1,2
5,10,
50

Appendix C

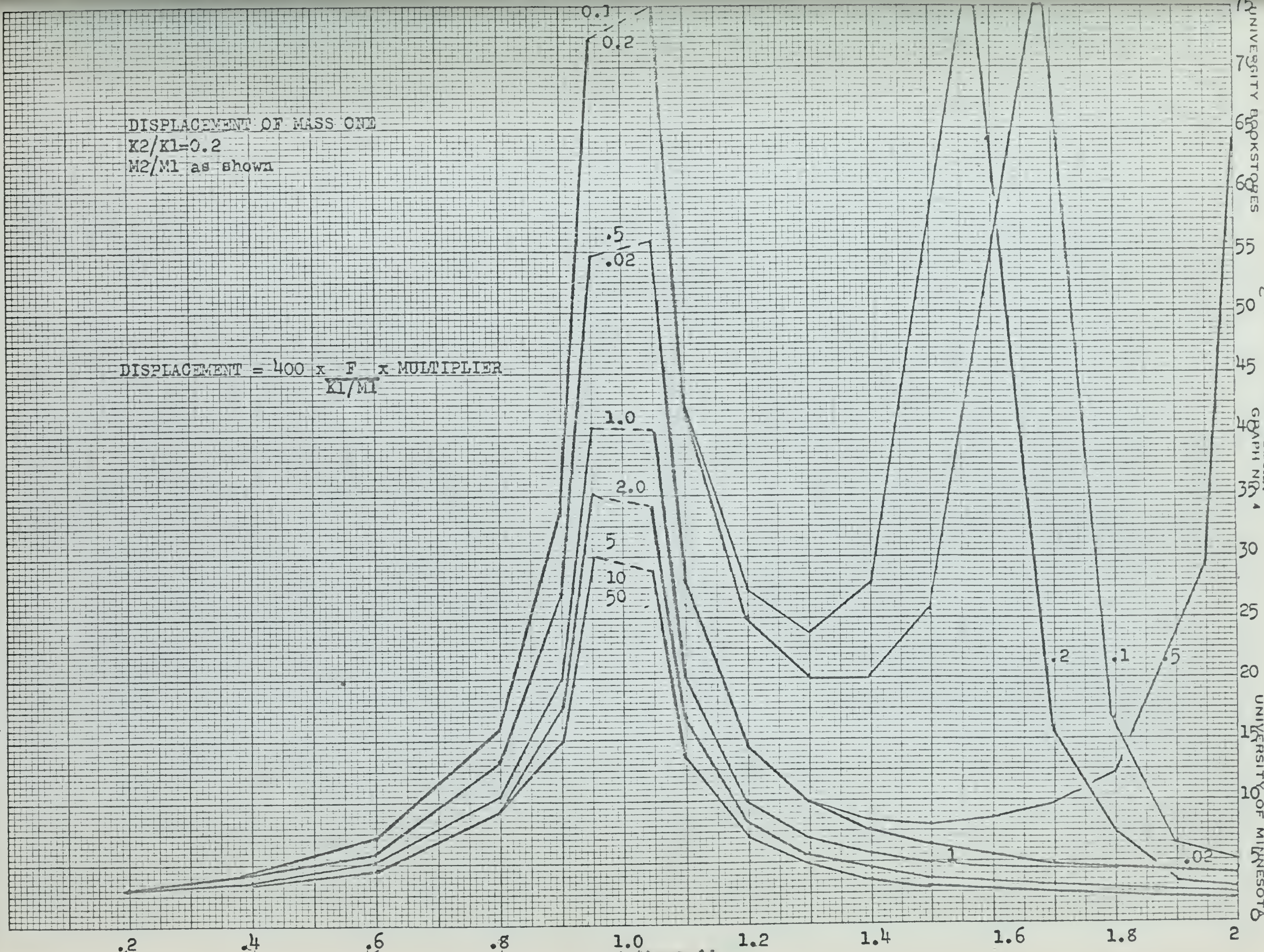


Figure 14

Appendix C

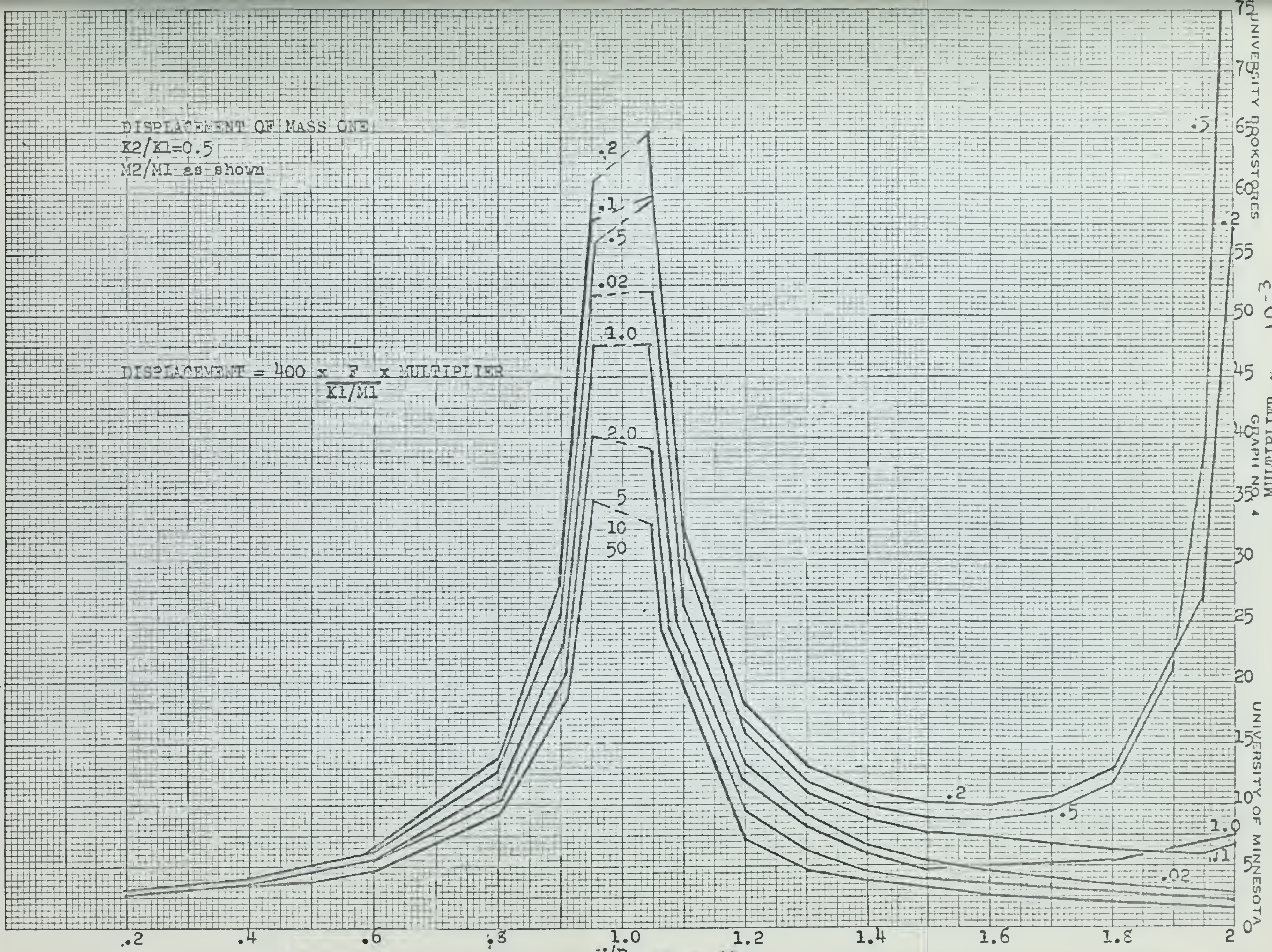
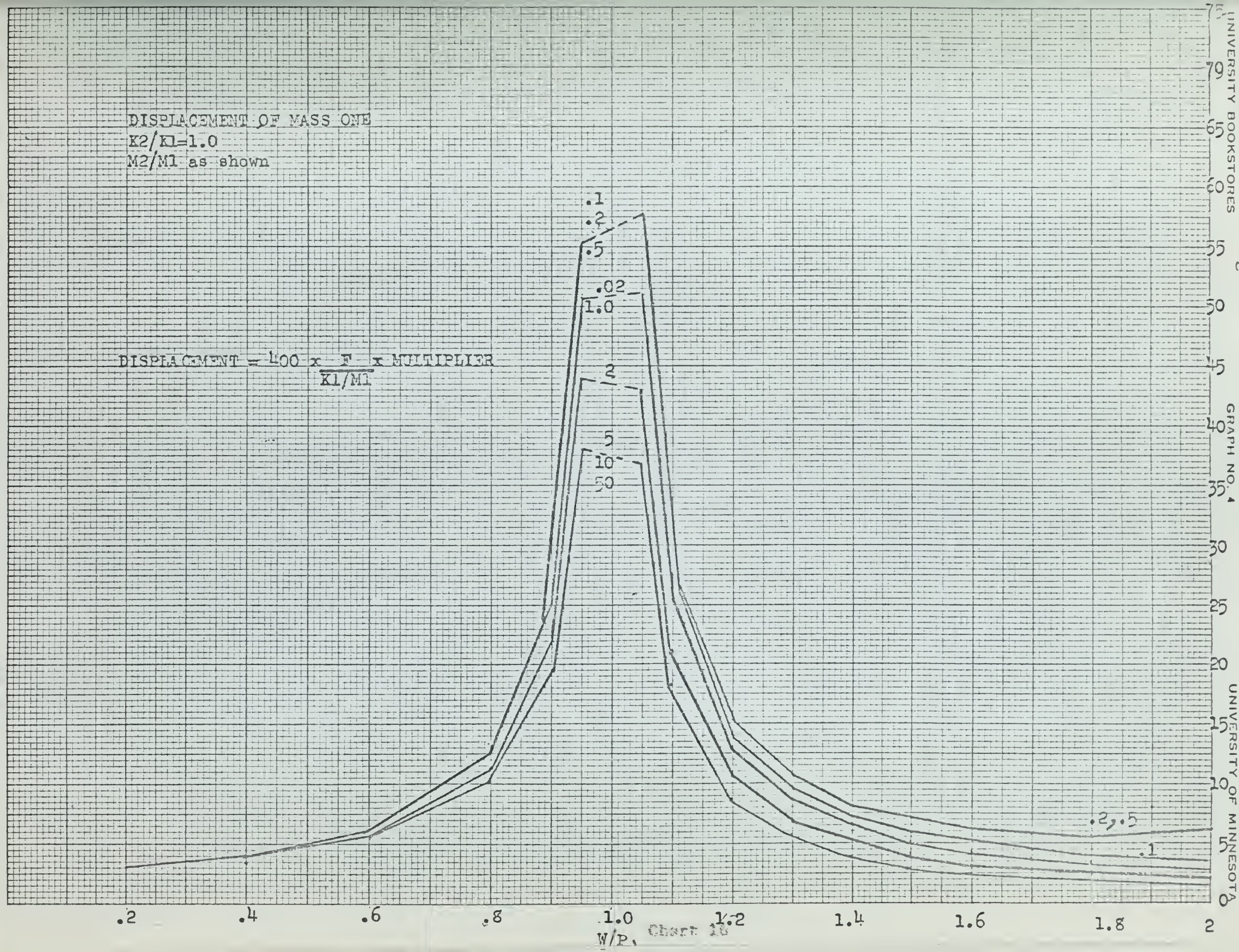


Chart 15

Appendix C



DISPLACEMENT OF MASS ONE
K2/K1=1.0
M2/M1 as shown
DISPLACEMENT = 400 x F/(K1/M1) x MULTIPLIER
.1
.2
.5
.02
1.0
2
5
10
50
.2,.5
.1
MULTIPLIER x 10^-3
W/P,
Chart 16
GRAPH NO. 4
UNIVERSITY BOOKSTORES
UNIVERSITY OF MINNESOTA

Appendix C

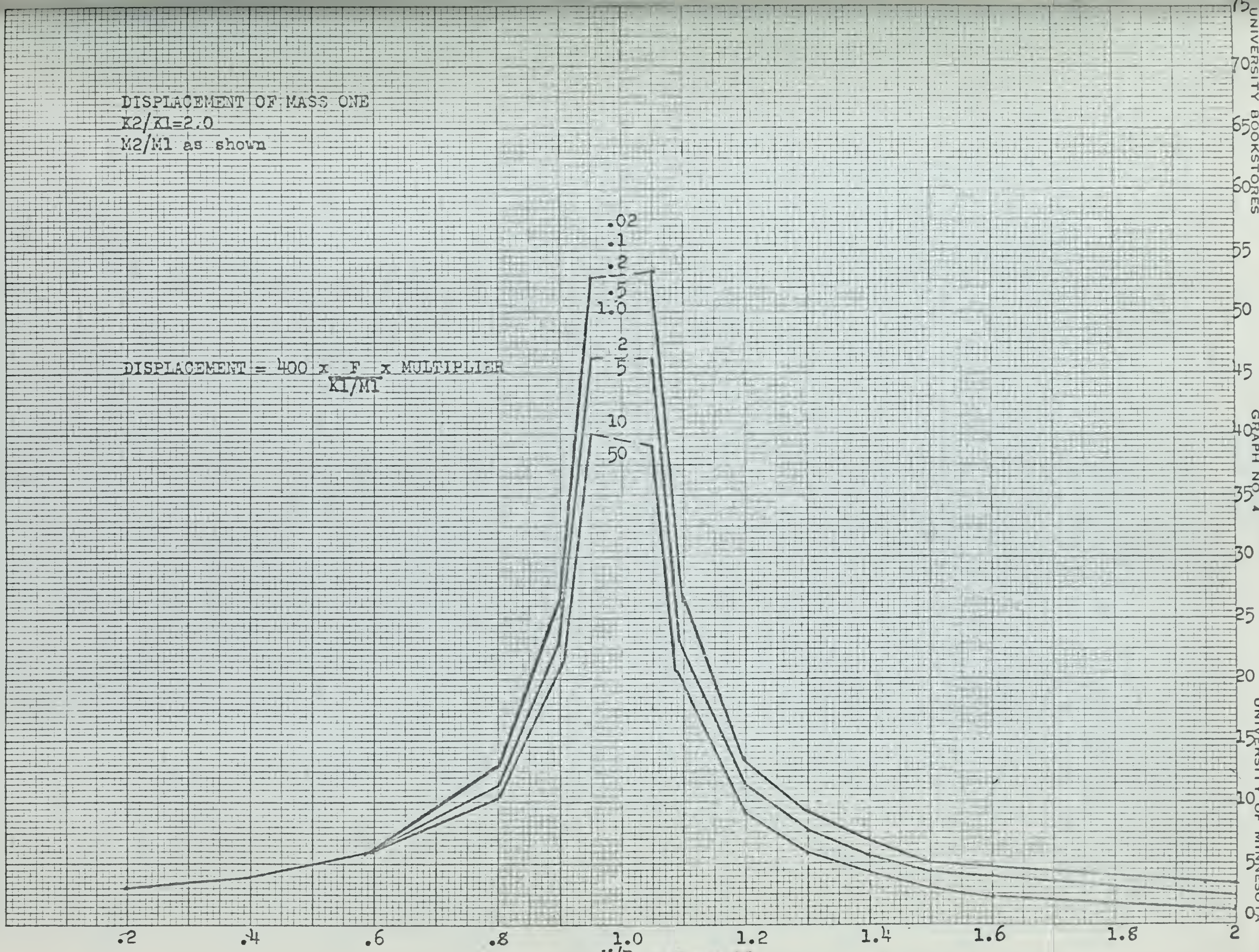


Chart 17

Appendix C

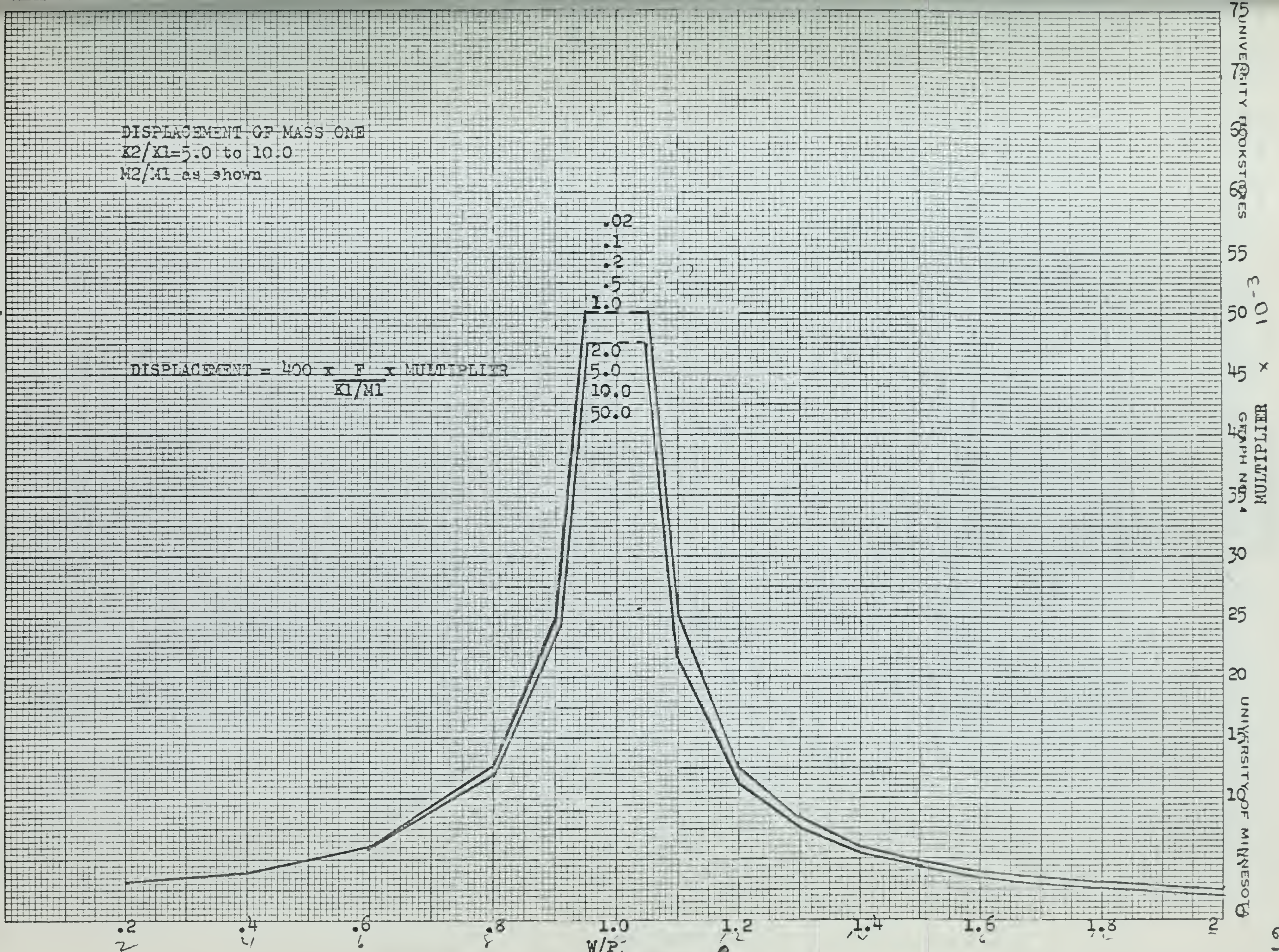
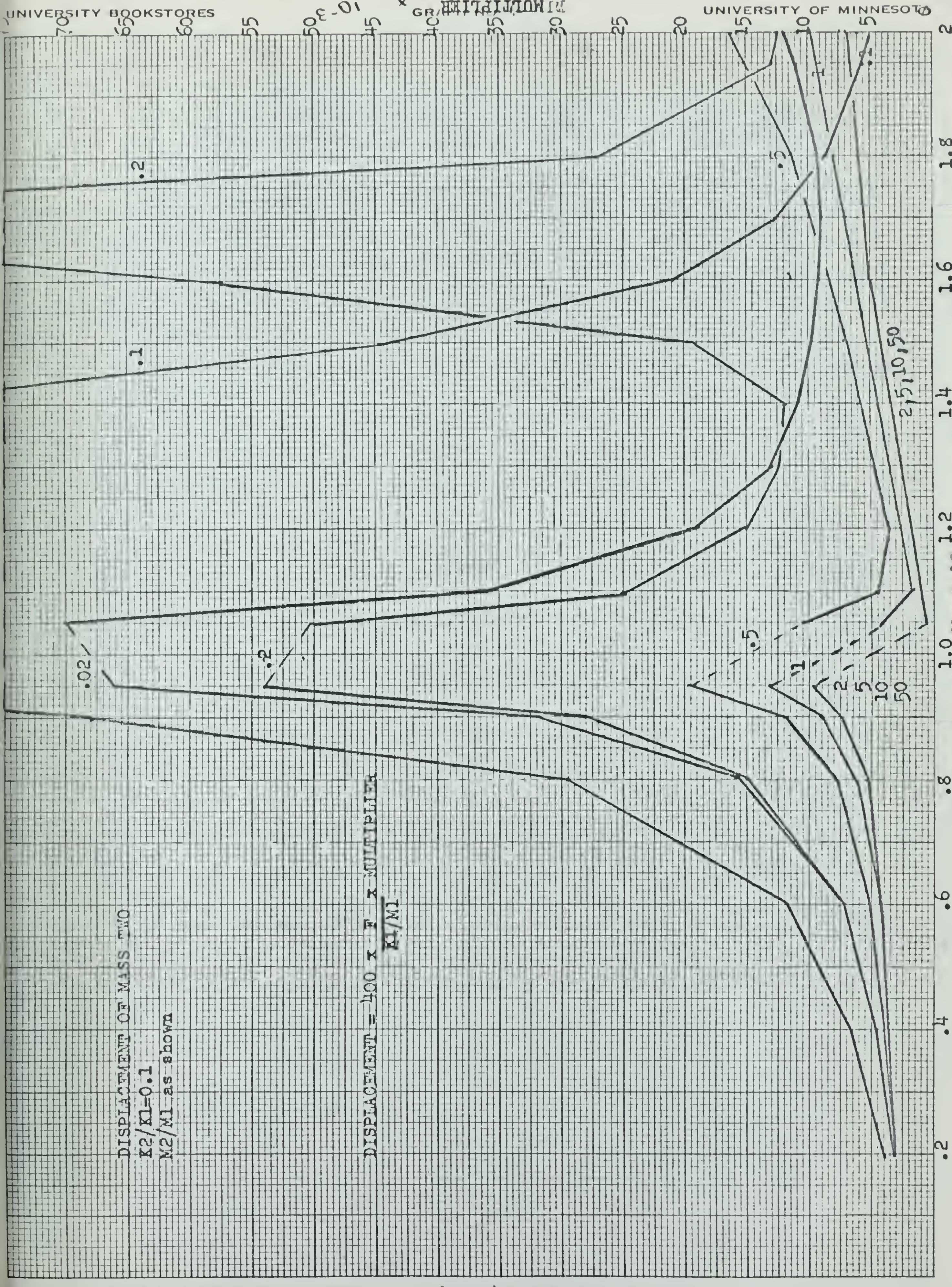


Chart 18

Appendix C


UNIVERSITY BOOKSTORES
UNIVERSITY OF MINNESOTA
DISPLACEMENT OF MASS TWO
K2/K1=0.1
M2/M1 as shown
DISPLACEMENT = 400 x F x MULTIPLIER / K1/M1
MULTIPLIER x 10^{-3}
W/P1
Chart 13
2,5,10,50

Appendix C

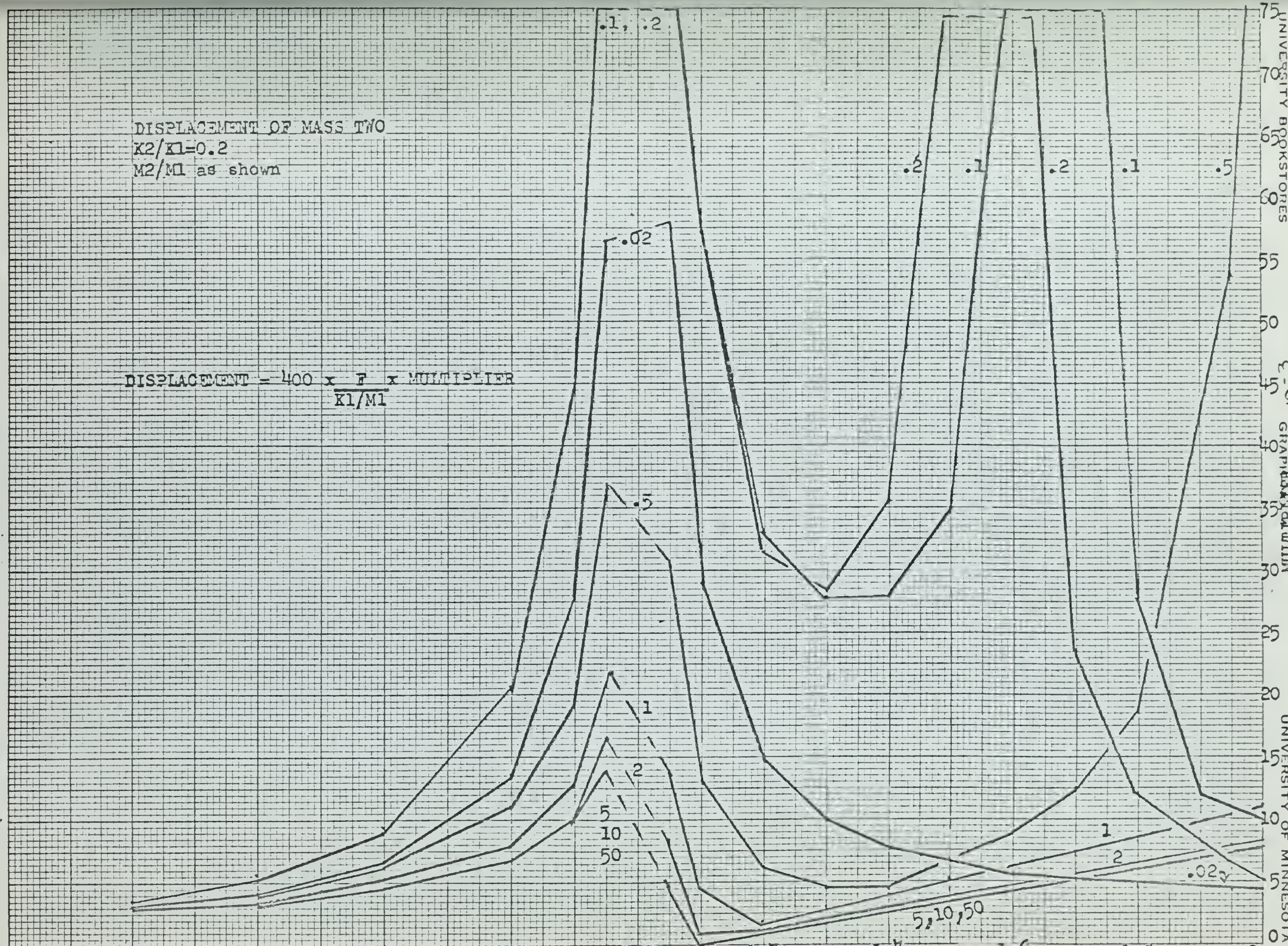


Chart 29

Appendix C.

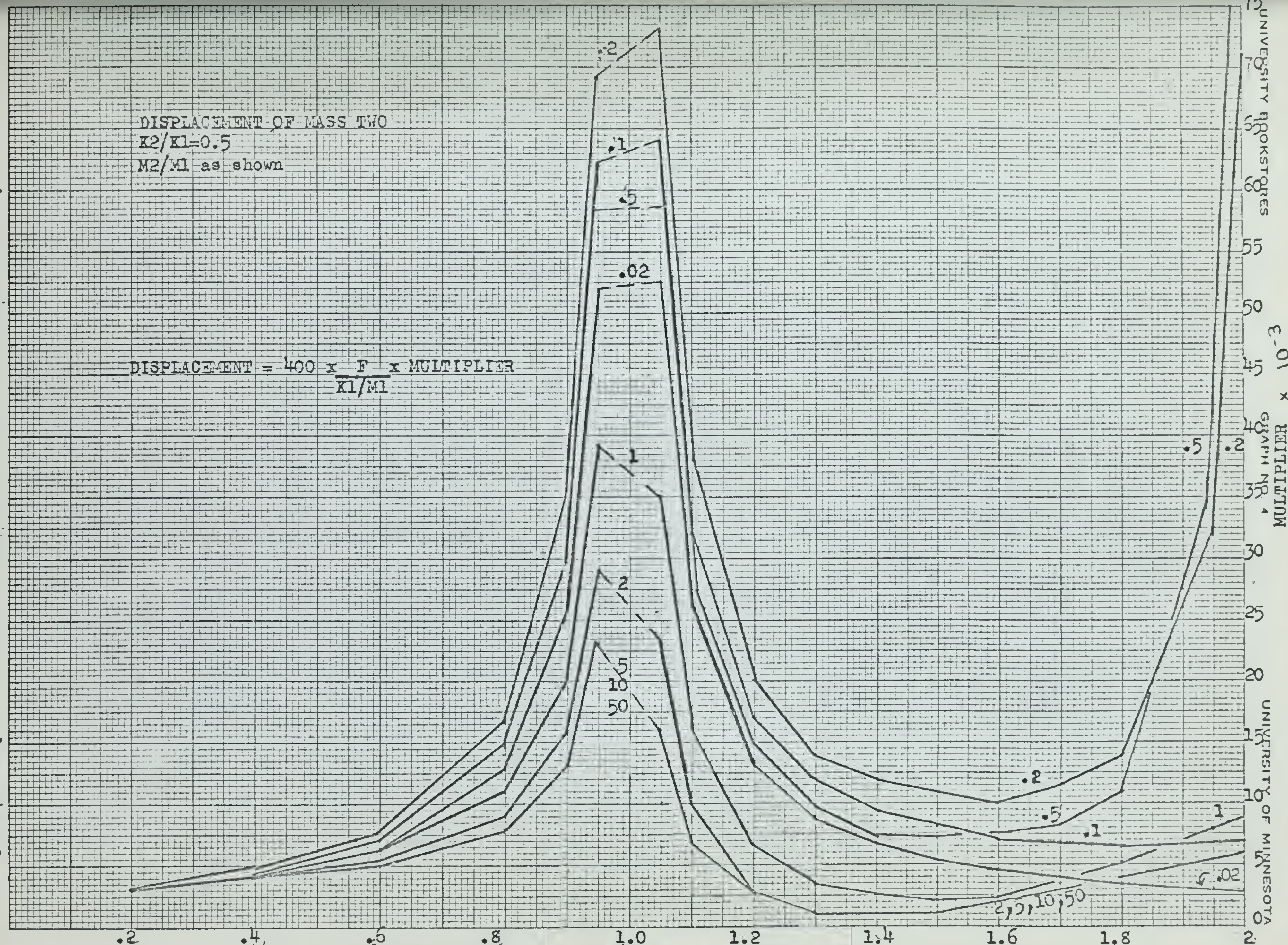
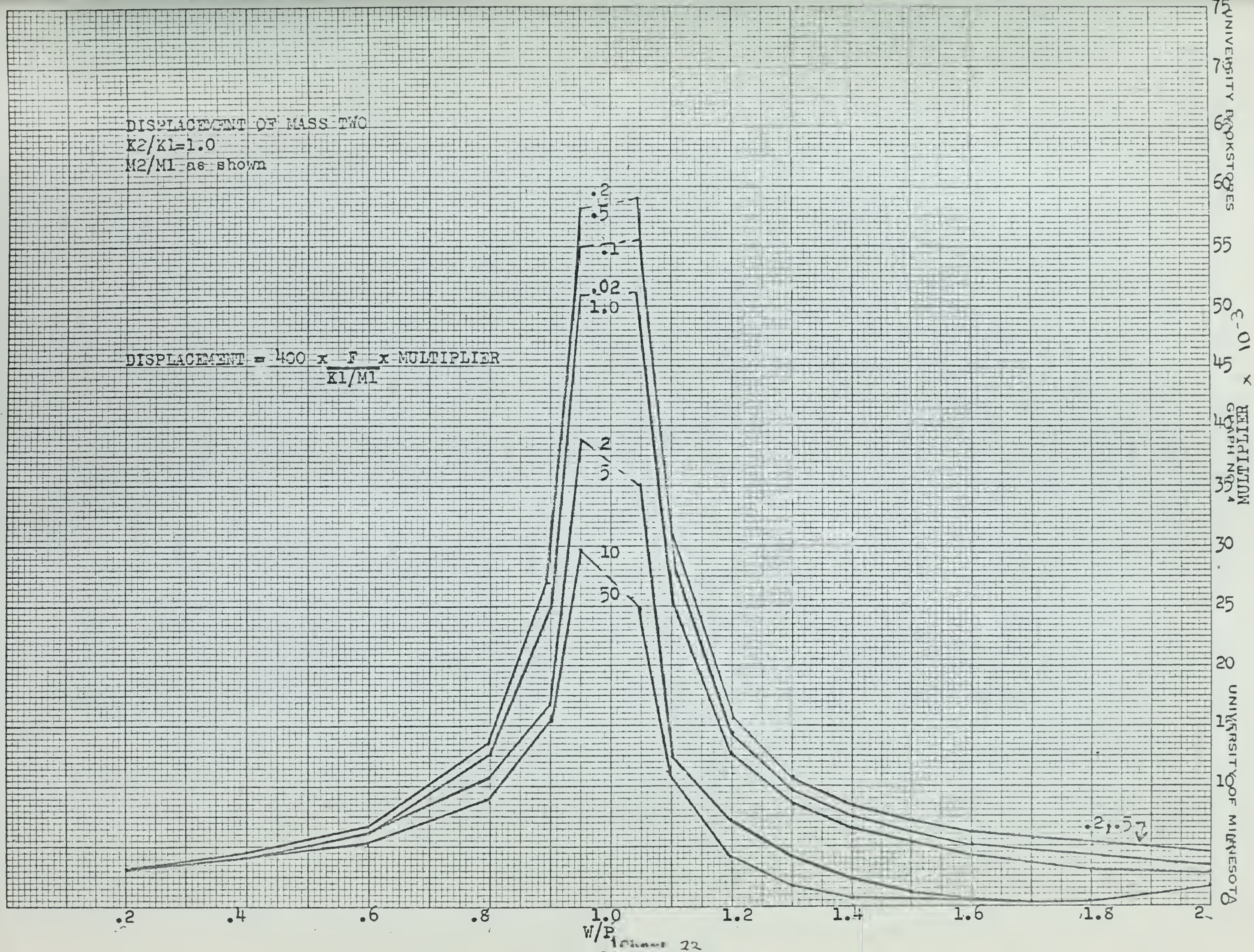


Chart 21

Appendix C


DISPLACEMENT OF MASS TWO
K2/K1=1.0
M2/M1 as shown
DISPLACEMENT = 400 x F / K1/M1 x MULTIPLIER
.2
.5
.1
.02
1.0
2
5
10
50
.2, .5
MULTIPLIER x 10-3
GRAPH NO 4
W/P1
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2
75
55
50
45
35
30
25
20
10
UNIVERSITY BOOKSTORES
UNIVERSITY OF MINNESOTA
Sheet 22

Appendix C.

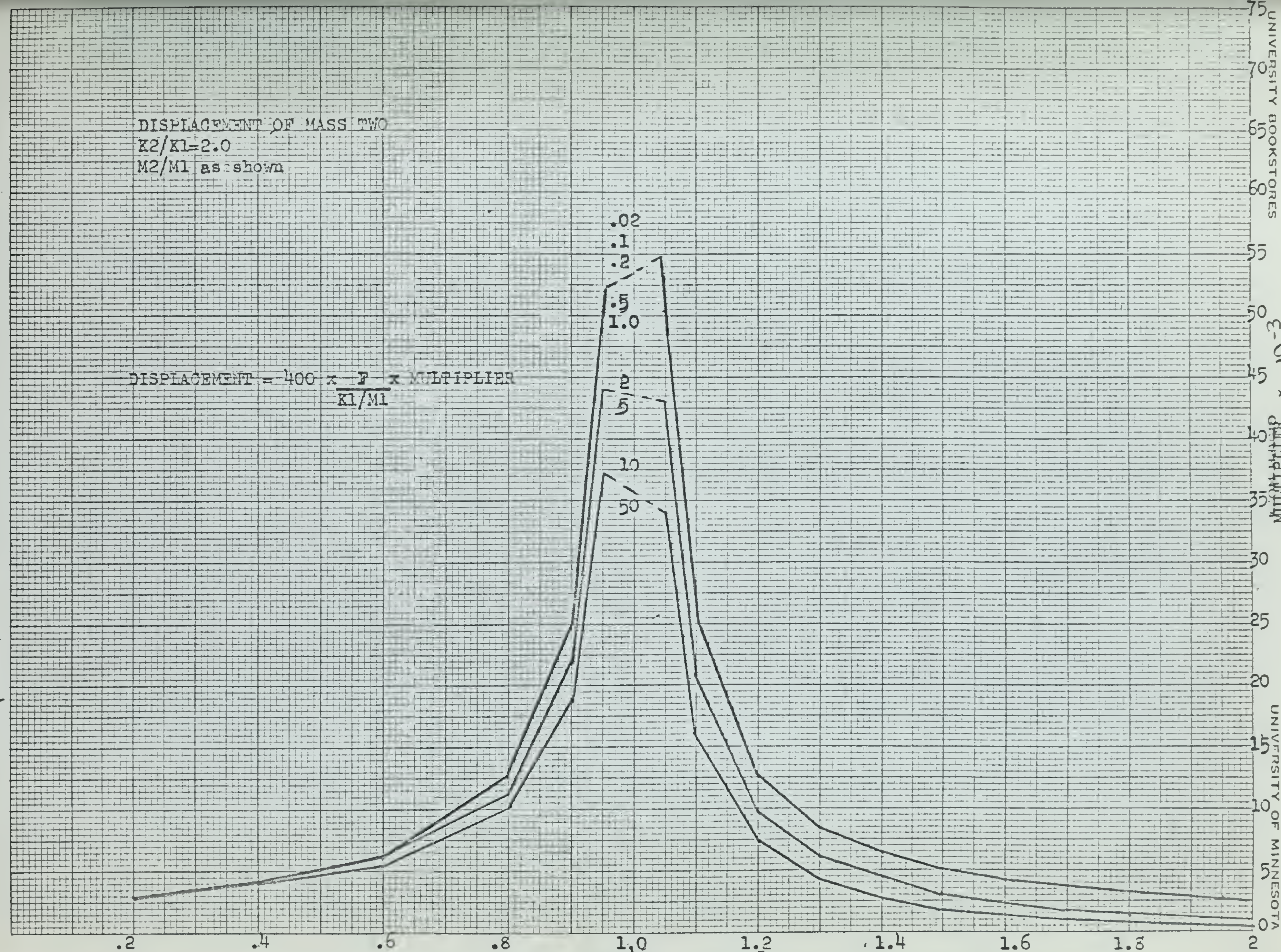
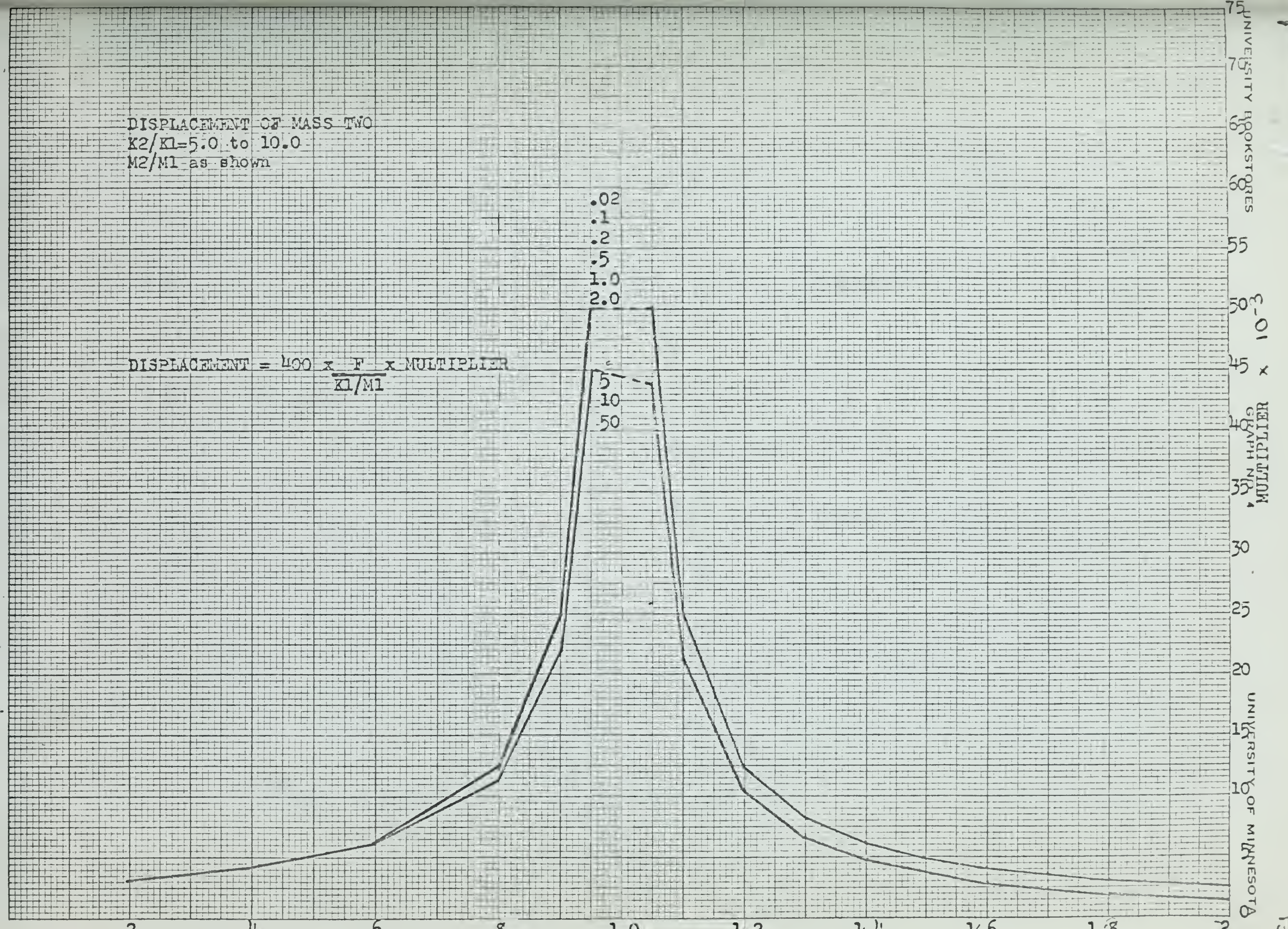


Chart 23

Appendix C


DISPLACEMENT OF MASS TWO
K2/K1=5.0 to 10.0
M2/M1 as shown
DISPLACEMENT = 400 x F/(K1/M1) x MULTIPLIER
.02
.1
.2
.5
1.0
2.0
5
10
50
MULTIPLIER x 10^-3
GRAPH NO. 4
UNIVERSITY BOOKSTORES
UNIVERSITY OF MINNESOTA
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2
W/P
Chart 24